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GEOMETRICAL AMUSEMENTS:

OR A COURSE OF

LESSONS,

IN

CONSTRUCTION AND ANALYSIS:

IN THREE PARTS.

BY J. H. SWALE.

PART I.

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Preface to Leslie's Elements.

LONDON:
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JOHNSON, PRINTER, LIVERPOOL.

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TO

J. LESLIE, Esq. F. R. S. E.

AND PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF EDINBURGH.

This Geometrical Trifle,

(THE FIRST PART OF THE AMUSEMENTS)

IS MOST RESPECTFULLY INSCRIBED:

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TESTIMONY OF ESTEEM FOR HIS WORTH,

AS A MAN;

AND FOR HIS DISTINGUISHED TALENTS,

AS A

GEOMETER;

BY THE

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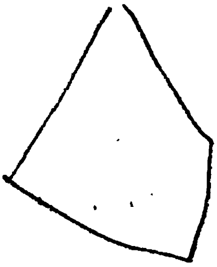

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Brunswick Place,
1st December, 1821.

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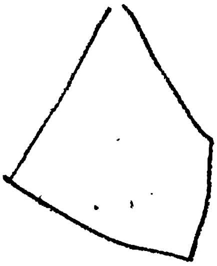

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BOOK I. CONSTRUCTION.

PROBLEM I. PAGE 11

A Point, and two right Lines, are given in position: to draw, through the given point, a right Line, meeting the Lines given in position, and cutting off, between the Points of Intersection and given Points in those Lines, Segments, having a given Ratio.

II. 13

To add, to a given Line, another Line, such, that the Rectangle under the compound Line, and the Segment added, shall be equal to a given Square.

III. 14

To determine two Lines; having given, their Rectangle, and the Difference of their Squares.

IV. 15

To describe a Circle, through two given Points, that shall touch a right Line, given in position.

V. 16

To a given Circle, to draw a Tangent, that shall be divided in a given Ratio, by the Point of contact, a Diameter given in position, and a Line perpendicular to that Diameter.

VI. 17

Given three Points, in the same right Line, to determine a fourth Point; such, that the Square of its Distance, from one of the given Points, shall have a given Ratio to the Rectangle under its Distances, from the remaining given points.

VII. 19

Given three Points, in the same right Line, to assign a fourth Point; such, that the Rectangle under its Distance, from one of the given points, and a given Line, shall have a given Ratio to the Rectangle under its Distances, from the two remaining given points.

VIII. 20

Given the Perimeter, the Line bisecting the vertical Angle, and the Difference of the Angles at the Base; to determine the Triangle.

IX. 23

Given the Sum, or the Difference, of a Line, to which the Base has a given Ratio, and a Line drawn from the Vertex, to make a given Angle with the Base; the Difference of the Segments of the Base, made by the Line from the vertex, and the Difference of the Angles at the Base; to determine the Triangle.

X. 25

Given in position, a Point, and four right Lines: to draw, through the given point, a right line, meeting two of the lines given in position: so, that drawing, from the Points of Intersection, two Lines, making given Angles with the two remaining lines given in position: the Sum, or the Difference, of two Lines, to which the lines, last drawn, have given Ratios, shall be given.

XI. 28

Two right Lines, and two Points, are given in position: to draw, through the given points, two parallel Lines, meeting the Lines given in position, and cutting off, from those Lines, Segments, having a given Ratio.

XII. 31

Three right Lines, and a Point, are, any how, given in position: to draw, through the given point, a right Line; so, that the Segments thereof, intercepted by the lines given in position, shall have a given Ratio to each other.

XIII. 33

Given the Chord and the Tangent, to assign the circular Arc.

XIV. 37

Having four Points given, in the same right Line: to determine a fifth Point; such, that the Rectangle of its Distances, from the first and third given points, shall have a given Ratio to the Rectangle of its Distances, from the second and fourth,

XV. 40

Given the Sum, or the Difference, of the Hypothenuse and one Side, and the Segment of the Hypothenuse, adjacent to that Side, by a Perpendicular from the right Angle, to determine the Triangle.

XVI. 43

Given, in position, a Point, and two right Lines: to draw, from the given point, two Lines that shall include a given Angle, and cut off, from the lines given in position, Segments, having a given Ratio.

XVII. 46

Two Points, and two right Lines, are given in position: to draw, from the given points, two Lines, including a given Angle, and cutting off, from the Lines given in position, Segments, having a given Ratio.

XVIII. 48

To describe, through a given Point, a Circle, to touch two right Lines given in position.

XIX. 50

To describe a Circle, through a given Point; such, that Parts of the Circumference, subtending given Angles, may be cut off, by two right Lines given in position.

XX. 52

To describe a Circle, that shall touch two right Lines given in position, and also a Circle, given in position and magnitude,

XXI. 54

Given in position, three Points, and two right Lines: to draw, from two of the given points, two Lines, parallel to each other, meeting the Lines given in position; so, that the two Points of concourse, and the remaining point given in position, shall be in the same right Line.

XXII. 58

Two right Lines, and a Point, are given in position: to draw, from the given point, two Lines, that shall intercept a Segment, of a given Length, on one of the Lines given in position, and cut off, from the remaining Line given in position, Segments, having a given Ratio.

XXIII. 60

Given in position, a Point, and four right Lines, two, of which, are parallel: to draw, from the given point, two Lines, intercepting, on one of the lines given in position, a Segment, of a given Length, and meeting the parallel lines given in position; so, that the right Line passing through the Points of Intersection, shall be parallel to the remaining line given in position.

XXIV. 63

Two Points and two right Lines, are given in position: to draw, from one of the given points, two Lines, including a given Angle, and meeting the lines given in position; so, that the Points of concourse, and the remaining point given in position, shall be in the same right Line.

XXV. 67

Given in position, three Points, and two right Lines: to draw, from two of the given Points, two Lines, including a given Angle, and meeting the lines given in position, in two Points; such, that a Line, passing through those points, shall also pass through the remaining point given in position.

XXVI. 71

Three right Lines, and two Points, are given in position: to draw, from the given points, two lines, having their Point of concourse in one of the Lines given in position, and cutting off, from the two remaining lines given in position, Segments, from given Points in those lines, that have a given Ratio.

BOOK II. ANALYSIS.

PROBLEM I. 83

Given in position, two right Lines, and a Point: to draw, through the given point, a right line, meeting the lines given in position, and cutting therefrom, Segments, the Rectangle of which shall be given.

II. 87

Given in position, a Line, at right Angles to the Diameter of a given Circle, and a Point, in that diameter, or in the diameter produced: to determine a Point, in the Periphery of the Circle; such, that, thence, drawing two right lines, one to the point given in position: the other meeting the line given in position, and making a given Angle therewith; the lines, so drawn, shall have a given Ratio.

III. 91

In the same right Line, three Points are given : to determine two other Points in the line; such, that the Distance of the points so determined, shall have a given Ratio to the Distance between one of them, and one of the given points : also, that the Square of the Distance, of the same determined point, from another of the given points, shall have, to the Rectangle under a given line, and the Distance of the other point to be determined, from the remaining given point, a given Ratio.

IV. 94

To assign the position of a Tangent, to a given circular Arc; such, that the Part of the Tangent, intercepted by Radii, through the Extremities of the given Arc, shall be of a given length.

V. 96

Given in position, a Diameter of a given Circle, a Point, in that diameter, or in the diameter produced, and a right Line, perpendicular to it: to determine a Point in the Periphery; such, that drawing, thence, a Line to the given point, and, also, a line making a given Angle with the line given in position; the Sum, or the Difference, of the lines thus drawn, shall be given.

VI. 100

Given in position, the same as in Problem V : to determine a Point, in the Periphery of the Circle; such, that drawing, thence, a Line to the given point, and another line, making a given Angle, with the line given in position : the Sum, or the Difference, of a given line, and the line drawn to the given point, shall have a given Ratio, to the line drawn, making a given Angle, with the line given in position.

VII. 103

Given in position, three right Lines and a Point, in one of them; also, another Point, any how posited : to draw, from the latter point, two lines, to cut off, from the line in which is the given point, two

Segments, between the Points of Intersection and the given point, having a given Ratio; and, also, to cut off, from the two remaining lines given in position, two Segments, between the Points of Intersection, and the Point of concourse of those lines, having to each other, another given Ratio.

VIII. 106

To draw, from two given Points, two Lines, intersecting each other at a Point, in a right line given in position, and cutting off, from another line given in position, two Segments, between the points of Intersection, and a given Point, in the line, of which, the Rectangle shall be given.

IX. 108

Three Lines and a Point, are given in position: to draw, from the given point, a right line intersecting two of the given lines, and cutting off, from one of them, a Segment, from a given Point therein; such, that joining the remaining point of Intersection, and a given Point, in the third line given in position, and drawing, from the first Intersection, a Parallel to the line joining those points: the Segment intercepted on the third line given in position, between a second given Point therein, and the point of Intersection of the Parallel, shall have a given Ratio to the Segment, cut from the other line given in position.

X. 111

Given, the vertical Angle, the Sum, or the Difference, of one Side, and a line, to which the other Side has a given Ratio; and the Ratio of the Segments of the Base, by a line from the Vertex, making a given Angle with the Base; to determine the Triangle.

XI. 113

The position of five right Lines, and of a Point, is given: to determine, in one of the lines, a point; such, that drawing, thence, two right lines, parallel to two other lines, also, given in position, intersecting the four remaining lines given in position: the right lines, joining two of the Points of Intersection, shall be parallel to the line, passing through the two remaining Points of Intersection.

XII. 116

Three right Lines are given in position: to assign the position of a Triangle, given in Species; such, that its angular Points shall be posited on the Lines given in position; and, that Segments, having a given Ratio, shall be intercepted on two of the lines, between given Points in those lines, and the angular Points of the Triangle.

XIII. 119

Four Points, and two right Lines, are given in position: to assign the position of Parallels, from two of the given points, meeting the lines given in position; so, that Lines, drawn from the Intersections, to the two remaining points given in position, shall be parallel.

XIV. 121

Three Circles being given in position and Magnitude: to describe a fourth Circle, intersecting the others; so, that two of the Chords, joining the Points of Intersection, shall pass through given Points, and the remaining Chord, be parallel to a Line given in position.

XV. 125

Given, the vertical Angle, the Radius of the inscribed Circle, and the Sum, or the Difference, of two Lines, to which the Base, and a Line, from the Vertex, making a given Angle with the Base, have given Ratios; to determine the Triangle.

XVI. 129

To describe, through two given Points, a Circle, intersecting a right Line given in position; so, that lines, drawn from either of the given points, to the Points of Intersection, shall contain a given Angle.

XVII. 132

Having four right Lines, given in position: to determine a Point, from which, if four other right Lines are drawn, meeting those given in position, in given Angles; the Sum, or the Difference, of two of the Lines, so drawn, and the Sum or the Difference, of the Rectangles, under the remaining two, and two given Lines, shall be given.

XVIII. 135

From two given Points, in the Diameter of a given Circle, to draw two Lines, to a Point, in the Periphery; such, that the Difference, or the Sum, of one of them, and a given Line, shall have a given Ratio to the other.

XIX. 138

Three right Lines, and three Points therein, are given in position; to determine three other Points, in the Lines; such, that the Segments of the lines, intercepted between the given points, and the points to be determined, shall have given Ratios to each other; and Lines, drawn from one of the determined points, to the remaining two, shall include a given Angle.

XX. 141

Given in position, four right Lines: to draw, parallel to another Line given in position, a Line, meeting the four Lines given in position; so, that the Rectangle, under two of the intercepted Segments, shall have a given Ratio to the Square of the remaining intercepted Segment, of the Line so drawn.

XXI. 144

To inscribe, in a given Triangle, another Triangle; such, that each Side, of the inscribed figure, shall pass through a given Point.

XXII. 146

To draw, from two given Points, in the Diameter of a given Circle, two right Lines, to a Point, in the Periphery; such, that the Difference, or the Sum, of those lines, shall be given.

XXIII. 150

From two given Points, any how posited, in the Diameter of a given

Circle, to draw two right Lines, intersecting at a Point, in the Periphery; such, that the Sum, or the Difference, of the Rectangles, under these two Lines, and two given lines, shall be equal to a given Magnitude.

XXIV. 153

Given, the Difference of the Angles at the Base, the Difference, or the Sum, of the Sides, and the Sum, or the Difference, of a Line, from the Vertex, making a given Angle with the Base, and a Line, to which the Difference of the Segments of the Base, by the Perpendicular, has a given Ratio; to determine the Triangle.

XXV. 157

Three Points, and two right Lines, are given in position: to draw, from one of the given points, two Lines, including a given Angle, and meeting the lines given in position; so, that Lines, drawn from the Points of Intersection, to the two remaining points given in position, shall include another given Angle.

XXVI. 160

Two right Lines, and four Points, are given in position: to draw, from two of the given points, two Lines including a given Angle, and meeting the lines given in position; so, that Lines, drawn from the Points of Intersection, to the two remaining points given in position, shall also include a given Angle.

XXVII. 164

Given in position, two right Lines, and a Point: to draw, from, the given point, a line that shall cut off, from the lines given in position, two Segments, between the Points of Intersection and given Points in those lines; such, that the Sum, or the Difference, of two lines, to which those Segments have given Ratios, shall be given.

XXVIII. 167

To draw, from two given Points, two Parallels, meeting two Lines given in position, and cutting therefrom, two Segments, between the Points of Intersection and given Points in the lines; such, that the

Sum, or the Difference, of two Lines, to which the Segments have given Ratios, shall be equal given a line; or, that the Rectangle of the two Segments cut off, shall be equal a given Space.

XXIX. 173

To draw, from two given Points, two Lines, including a given Angle, meeting two Lines *given* in position, and cutting therefrom, two Segments, between the Points of Intersection, and given points in the lines; such, that the Sum, or the Difference, of two Lines, to which the Segments cut off, have given Ratios, shall be given; or, that the Rectangle of the Segments shall be equal a given Square.

XXX. 179

Given in position, three right Lines, and two Points: to draw, from the given points, two Lines, having their Point of concourse in one of the lines given in position, meeting the two remaining lines given in position, and cutting therefrom, two Segments, between the Points of Intersection and given points in the lines; such, that the Sum, or the Difference, of two Lines, to which the Segments have given Ratios, shall be given; or, that the Rectangle of the Segments, cut off, shall be given in Magnitude.

BOOK III.

GEOMETRICAL EXERCISES.

Problem I. to Problem CXXXVI. Page 191 to 236.

PART II. and III., of the Amusements, are ready for the Press; and will appear in Succession.

ERRATA.

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72	11	Bottom	QLR	QKR
86	7	Top	AR : BH	AR—BH
136	8	Bottom	PR : GN	GN : PR
140	3	Top	HEK	GHK
151	5	Bottom	QR : QL	QR : RL

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GEOMETRICAL

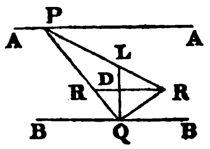
AMUSEMENTS.

GENERAL PROBLEM.

Having two Points and three right Lines, given in position: to draw, from the given points, two right lines, intersecting each other at a point, in one of the lines given in position, and making equal angles with the two remaining lines, given in position.

This Problem divides itself into different cases, depending upon the relative situation of the points and right lines, given in position.

A.



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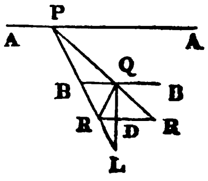


1. In figure A, the three lines, PA, QB, DR, given in position, are parallel to each other, and the given points P, Q, on contrary sides of DR, the line in which the two lines, required to be drawn, are to intersect each other.

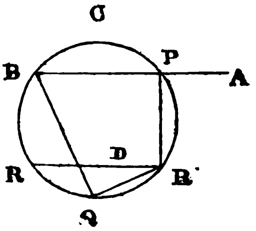
In B, the lines are parallel, but the points P, Q, are on the same side of DR.

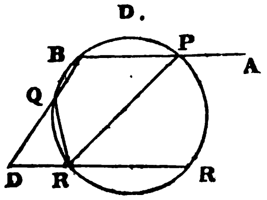
From one of the points, as Q, draw QD perpendicular to DR, make DL = DQ, through P, L, draw PR, meeting DR in R, and join Q, R: the required lines are PR, QR.

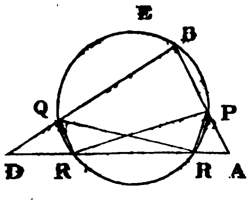
For, DQ = DL, and DR is perpendicular to QL, or the angle LRD = angle QRD: and because of parallel lines, the angle LRD = angle RPA, and the angle QRD = angle RQB′: or the angle RPA = angle RQB.

In this case of the problem, a point R, answering the conditions, may be determined, by drawing PQ, intersecting DR in R.

For obviously, because of parallels, the angle RPA = angle RQB.







2. In figure C, the lines PA, DR, only, are parallel, and the points P, Q, on contrary sides of DR.

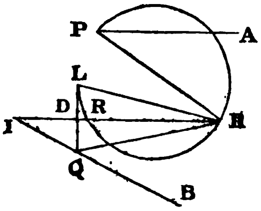
In D, the lines PA, DR, are parallel, but P and Q, are on the same side of DR.

In E, the lines are any how posited, and P, Q, on the same side of DR.

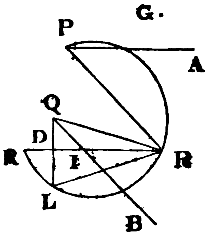
Let Q B meet PA at B, through P, Q, B, describe a circle, cutting DR in R, the point to which PR, QR, are to be drawn.

For, the points P, Q, R, B, being in the periphery of a circle, it follows, that the angle RPA=RQB.

F.



G.



3. In figures, F, G, the lines and points are posited as in figures C, D, but the angles APR, BQR, are both exteriorly situated.

Let QB meet DR at I: draw QD perpendicular to DR, make DL = DQ, and through P, L, describe a circle to contain the angle QID: this circle will intersect DR at R, the point to which

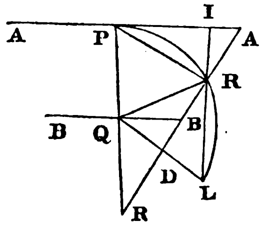
PR, QR, are to be drawn.

Draw LR.

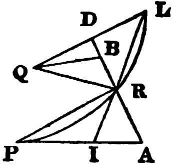
Since QD= DL, and DR perpendicular to QL, we have the angle QRI = LRI: also, by the construction, the angle PRL = QID = the difference or the sum, of the angles QRI, RQB, or of the angles QRI, IRP, or of the angles QRI, RPA, because of parallel lines:

That is, the angle RPA = RQB.

H.



I



4. In figure H, the lines PA, QB, are parallel, and P, Q, on the same side of DR.

In I, the lines are any how posited, and the points P, Q, on the same side of DR.

Draw QD perpendicular to DR, make DL = DQ, and describe, through P, L, a circle to contain the supplement of the difference of the angles QBR, PAR: the point R, and the position of the lines PR, QR, are determined as in case 3.

Draw LR, meeting PA at I.

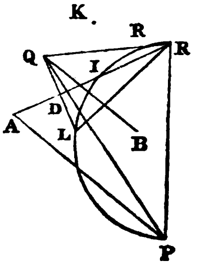
As in the preceding, the angle LRD = QRD; and, by the construction, the angle PRI = the difference of the angles, QBR, PAR, or of the angles PRA, IRA, or of the angles PRA, LRB: that is, equal the difference of the angles PRA and QRB.

Therefore, the sum of the angles QBR, QRB, is equal to the sum of the angles, PAR, PRA.

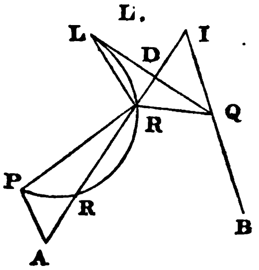
And consequently, the angle RPA = RQB.

In figure H, draw PQ, and produce it to DR at R, which is a point having the property required by the problem, as in figures A and B.

K.



L.



5. In figure K, the lines PA, QB, are parallel, and P, Q, on contrary sides of DR.

In L, the lines are any how posited, and P, Q, as in figure K.

Let QB meet DR at I; draw QD perpendicular to DR, make DL = DQ: a circle through P, L, to contain an angle equal to the difference of the angles QIR, PAR, in figure K; and to their sum, in figure L, determines R, and, thence, the position of PR, QR.

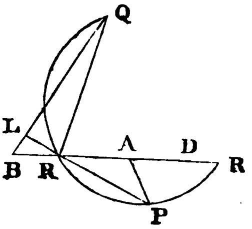
Draw LR.

We show, as before, that the angle QRI= LRI: also, by the construction, the angle PRL=the difference or the sum, of the angles QIR, PAR, equal the difference of the angles PRI, QRI: Or the sum of the angles QIR, QRI, is equal the sum or the difference, of the angles PRI, PAR.

That is, the angle RPA= RQB.

When the lines and points are situated as in figure K, draw PQ intersecting DR in R, a point answering the conditions of the problem: for, by parallels, the angle RPA= RQB.

M.



6. In figure M, the lines are any how posited, and the points P, Q, on contrary sides of DR.

Let PA, QB, meet DR, at A, B.

Through P, Q, describe a circle to contain the supplement of the difference of the angles BAP, ABQ: the intersection of this circle with DR, determines R.

Draw PR, QR, and let PR meet QB at L. By the construction, the angle QRL = the difference of the angles BAP, ABQ, or of the angles QRB, LRB, or of the angles QRB, ARP.

Therefore, the sum of the angles BAP, ARP, is equal the sum of the angles ABQ, QRB: That is, the angle RPA = RQB.

It would be superfluous to give the construction for any case of the problem, where the lines PA, QB, are supposed not to pass through the points P, Q, but to have some other assigned position: since, in every possible position of those lines, parallels to them may be drawn, through the points P, Q; and the case under consideration, thus rendered coincident, with the corresponding one of the preceding. From the above method of considering this problem, it appears that,

If two lines, drawn from given points to have their point of intersection in a right line given in position, make equal angles with two other lines given in position: the lines so drawn shall either include a given angle:

As in cases 2. 6. and figures C, D, E, M.

Or,

One of them shall pass through a given point:
As in case 1. figures A, B.
Or,
Shall make a given angle with a line passing through a given point.
As in cases 3. 4. 5. and figures F, G, H, I, K, L.

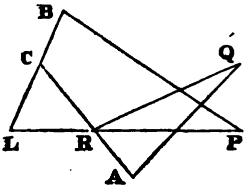
As the foregoing constructions apply to all the material cases, we shall now show the Use of the Problem, in its application to the solution of a *Series of Geometrical Inquiries.*

BOOK I.

CONSTRUCTION:

PROBLEM I.

The point P, and the right lines CA, CB, are given in position: to draw the right line PRL, meeting CA, CB, at R, L, so, that the segments AR, BL, shall have a given ratio.



CONSTRUCTION.

Draw PB, AQ, making the angle CAQ=PBC, and AQ : BP=the given ratio: then, by the *General Problem*, draw PR, QR, so, that the angle BPR=AQR.

DEMONSTRATION.

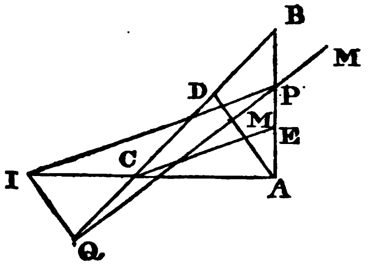
Let PR meet BC at L.

By the construction,

The angle QAR=PBL, and AQR=BPL;

Or the triangles QAR, and PBL, equiangular, having AR : BL=AQ : BP equal the given ratio.

'From the above, we obtain an extremely simple and elegant solution, to the XIX question, Leeds Correspondent: which is, 'To draw, through a given point M, a right line MPQ, meeting two of the sides BA, BC, of a given triangle ABC, in P, Q, so, that drawing PI, QI, parallel to lines given in position; the point of concourse I shall be in the remaining side AC of the triangle.'



Let CE, AD, be parallel to the lines given in position: and draw, by Prob. I, MPQ, making $AP : DQ = AE : CD$.

Draw PI parallel to EC, and join Q and I.

By the Construction,

$AP : AE = DQ : DC = AI : AC$, because of the parallels IP, CE:

Hence, dividing and alternating, $CD : CA = CQ : CI$;

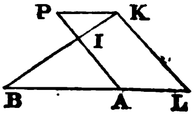
That is, QI is parallel to AD.

Problem I. which we may designate as our leading *Auxiliary Problem*, is the same as the general case of the Section of Ratio of the Ancients: for which we are indebted to the fertile genius of APOLLONIUS, the celebrated Geometrician of Perga.

I am not aware that any other problem, so generally applicable to composition, can be found in the compass of plane geometry. The above, with three of its particular cases, forms the first four Propositions of Book II. Professor Leslie's Geometrical Analysis: a work of considerable originality, ingenuity and address.

PROBLEM II.

To add, to a given line, another line such, that the rectangle under the whole line and the segment added, may be equal a given square.



CONSTRUCTION.

In any position, take BA=the given line, and BA . AP=the given square: draw PK parallel to BA, also BIK, by Prob. I. making AI=PK: the required line is PK.

DEMONSTRATION.

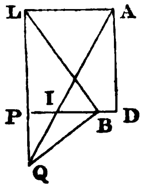
Draw KL parallel to PA.
Because of parallel lines and equals,
BA : AI=BL : LK=BA : PK=BL : AP=BA : AL
That is, BL . LA=BA . AP the given square.

If BA be taken as above, and AP=the side of the given square, draw BIK, making AI : PK=BA : AP.

This apparently trifling problem is of singular utility in construction, and has received the notice of most of the elementary Writers. The ardent Geometer will not be displeased with the attention I have paid to it, in the third part of these amusements.

PROBLEM III.

To determine two lines, having given their rectangle and the difference of their squares..



CONSTRUCTION.

Take AD the side of the square, equal the given difference, and AP=the given rectangle: then draw, by Prob. I. AIQ, making $DI : PQ = AD : DP$; apply to PD, $QB = PD$, and draw LB: the required lines are LB, BP.

DEMONSTRATION.

By the Construction, parallels, and equals,

$$AD : DI = DP : PQ = AL : PQ = QL : LA = QB : QP = QL : QB,$$

Or the triangles QBP, QLB, LBP, are equiangular, having

$$LB : BQ = LP : PB,$$

Or $LB \cdot BP = BQ \cdot LP = DP \cdot PL$ the given rectangle.

Also,

The angle LPB is a right angle,

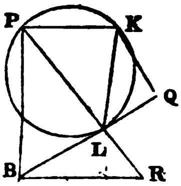
Or $LB^2 - BP^2 = LP^2 = AD^2$ equal the given square.

This problem, which so frequently presents itself to the Geometer, as the result of his Analysis, is, essentially, the same as the II.; since its construction obviously includes the determination of that problem: for $LP = AD$, is a given line, and PQ has been determined, making the rectangle $LQ \cdot QP = QB^2$ or PD^2, a given square.

In the subsequent pages of the Amusements, I have again noticed this geometrical trifle.

PROBLEM IV.

To describe a circle through two given points, that shall touch a right line given in position.



CONSTRUCTION.

Join the given points P, K; draw KQ perpendicular to BQ, the line given in position, also PB, BR, perpendicular to PK, BP: then, by Prob. I. let PLR be drawn, making QL : BR =KQ : PB: the point of contact is L.

DEMONSTRATION.

Describe a circle through P, K, L, and draw KL.

By the construction,

QL : QK=BR : BP,

Or the right angled triangles LQK, RBP, are equiangular,

Having the angle QLK=BRP=LPK, because of parallel lines;

Or BQ touches the circle at L:

That is,

The circle through P, K, touches BQ at L.

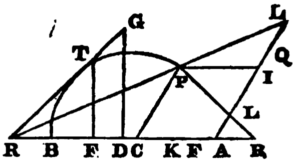
This problem is the same as, To determine two lines, of which, the rectangle and ratio are given.

Or, To draw, from two given points, two lines intersecting each other in a right line, given in position, and, with two other lines, also given in position, making angles, of which, the sum or the difference shall be given.

The identity and solutions of these problems, are left for the consideration and amusement of the student.

PROBLEM V.

To a given circle, to draw a Tangent, that shall be divided in a given ratio, by the point of contact, a diameter AB given in position, and a line DG, perpendicular to that diameter.



CONSTRUCTION.

In any position, parallel to each other, draw from C, the center, and A, the extremity of the diameter given in position, the radius CP, and AQ=AD: take CD : CK=the given ratio, and through P, by Prob. I. draw LPR, making QL : DR= CK : KD; the Tangent from R is that required.

DEMONSTRATION.

Draw the Tangent RTG, also PI parallel, and TF perpendicular, to AB.

Because of parallels,

RC : CP=PI : IL=CP : IL,

Or RC . IL $=CP^2=$ RC . CF :

That is, IL =CF.

Hence, QI being equal to CD, we also have QL= FD;

And RG : GT =RD : DF= RD : QL= DK : KC, by construction;

Therefore, RT : TG= CD : CK the given ratio.

We hence derive a solution to the following problem.

To determine two lines DR, DF, having to each other a given ratio, such, that adding a given line DC to each, the rectangle of the compound lines CR, CF, shall be equal to a given square.

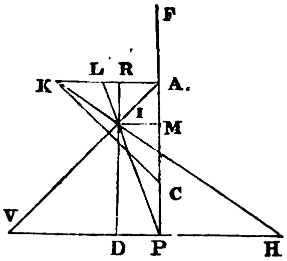
Among further Deductions from the preceding, we notice the following simple and beautiful Theorem.

From A, the point at which any right line LA meets a given circle APB, draw the diameter AB, also the radius CP, parallel to AL: assign, in AL, AB, any equal distances whatever, AL, AF, and draw FT perpendicular to AB.

Then, the Tangent to the circle at T, and the right line through L, P, will meet AB at the same point R.

PROBLEM VI.

In the right line AP, there is given the point C: to determine another point M, in AP, such, that the rectangle PM . MC, shall have a given ratio to the square of AM.



CONSTRUCTION.

Make the perpendiculars PV, AK, equal to PA, CA, and draw AV, CK: take FC : CA= the given ratio, and, by Problem I. draw KIH, making AI : PH= CK : AF; then IM, parallel to VP, determines M.

DEMONSTRATION.

Let PI meet AK at L, and draw, through I, DR parallel to PA.

By parallels, equals, and construction,

$$PM : MI = IR : RL = PM : MA = MA : RL,$$

$$\text{Or, } PM . RL = AM^2 :$$

$$\text{Also, } AI : PH = CK : AF.$$

Now, by equiangular Triangles,

$$AI : PD = CK : CA,$$

Then, $PH : PD = AF : AC = KL : LR$, by parallel lines:

Hence,

$$RK : RL = CF : CA = MC : RL = PM . MC : PM . RL$$
$$= PM . MC : AM^2$$

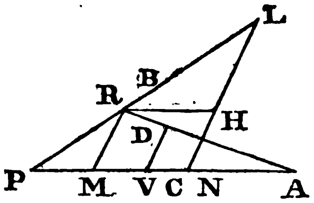
Equal the given ratio.

When the ratio is that of equality, the Construction becomes extremely simple: since, in this case, the points A, F, as also P, H, will be coincident; and, of course, KIH will pass through P: that is, IM, drawn from the intersection of AV, PK, determines M.

This Problem is the same as, To determine, in a right line ACP given in position, and intersecting a given circle in C, P; a point M, such, that drawing a Tangent to the circle, from M; the segment AM shall be equal to that tangent, or shall have any given ratio to it.

PROBLEM VII.

In the right line AP, there are given, the points C, N: to determine the point M, in AP, such, that the rectangle PC . PM shall have a given ratio to the rectangle MA . MN.



CONSTRUCTION.

Take AV= PC, draw the parallels VD, NL, make NL = NP, and join P, L : Let PB : PN= the given ratio, and draw, by Problem I. ADR, so, that VD : RL= PB : PL : the parallel RM determines M.

DEMONSTRATION.

Draw RH parallel to PN.

By parallels and equals,

AM : MR= AV : VD= AM : MP =PC : VD,

Or, MA . VD= MP . PC.

Also, by the construction,

VD : RL= PB : PL;

And because of parallel lines,

RH : RL=MN : RL=PN : PL:

Then,

VD : MN= PB : PN= MA .VD : MA . MN =MP . PC: MA . MN the given ratio.

The ratio of equality will be determined, by drawing ADR, making VD : RL= NP : PL.

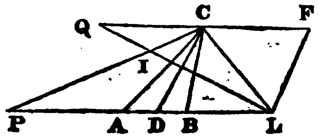
The last construction, besides furnishing us with a Solution to the problem deduced from the VI. applies to a more general one. To determine M, such, that the rectangle under the segment MP and a given line, shall be equal, or have any given ratio, to the square of the tangent from M, to any circle passing through the points N, A.

We have also solved the following, To draw, from a given point, a right line, meeting two right lines, given in position, and cutting off segments, such, that the sum or difference, of one of the segments, and a line to which the other segment has a given ratio, shall be given.

The present, and Problem VI. are cases of the *Determinate Section*, another Bequest of the GREAT GEOMETRICIAN of Perga.

PROBLEM VIII.

Given the perimeter, the line bisecting the vertical angle, and the difference of the angles at the Base, to determine the Triangle.



CONSTRUCTION.

Take LP, angle PLQ, and LF, perpendicular to LQ, equal the perimeter, half the given angle, and the bisecting line, respectively; and let FQ be parallel to LP: then drawing, by Problem I. PIC, making LI : QC= PL : LQ, the vertex of the required triangle is C.

DEMONSTRATION.

Draw CL, CA, CB, CD, making the angle PCA= CPA, LCB= CLB, and CD parallel to FL:

It is to be proved, that ACB is the required Triangle.

Because of equal angles, AC = AP, and BC = BL,

Or the sum of the sides AC, CB, BA, is equal to LP, the given perimeter.

Also, by the construction,

PL : LI = LQ : QC,

And the angle PLI = LQC;

Therefore, PLI, LQC, are equiangular triangles,

Or the angle LPI = QLC:

That is,

The difference of the angles CLP, CPL, is equal to the angle PLI:

And, as the angles at B, A, are, obviously, double of those at L, P,

It follows, that the difference of the angles B, A, is double the angle PLI:

Or equal the given difference.

Moreover, CD is perpendicular to LI,

And LI makes with the base BA, an angle equal to half the difference of the angles at B, A; or CD bisects the angle ACB:

Also, CF, DL; and CD, FL; are parallel, or CD = FL;

That is, CD is equal the given bisecting Line.

◆

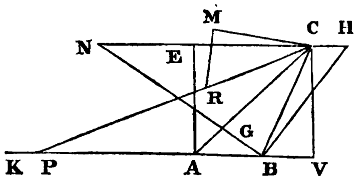
This is Problem LXI. Appendix, Simpson's Algebra, fourth Edition. The Data might have been varied, as, for the line bisecting the vertical angle, a line having a given ratio to it; or the perpendicular, or a line having a given ratio to the perpendicular: or, Given the perimeter, and two lines having given ratios to the perpendicular and the line bisecting the vertical angle, to determine the triangle.

To each of these Cases, the foregoing construction is applicable.

When the perimeter, the perpendicular, and the vertical angle, are given.

Take KV = the perimeter, perpendicular VC = CM = that given, angle VCM =the given angle with half its supplement, MR perpendicular to MC: and draw CRP, by Problem I. making MR= KP; then CA, making angle PCA= CPA, determines A, an angle at the Base.

This is the LX Problem Simpson's Appendix, it would be more general, by making the second Datum, a line making any given angle with the Base.



When the base, the perpendicular, and the difference of the angles at the base, are given, for the determination of the triangle ACB.

Make AB= the base, AE =the perpendicular, EC parallel to AB, ABN= half the given angle, NBH a right angle: Then draw, by Problem I. AGC, making BG : HC= 2 BN: NH : join C, B; the triangle is determined.

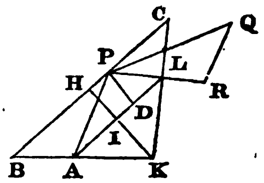
This is the XV Problem, Appendix of the same work.

Here again, we may diversify and generalize, as, Given the base, difference of the angles at the base, and either a line from the vertex, making any given angle with the base, or the line bisecting the vertical angle, or a line having a given ratio to the perpendicular, or to the bisecting line: or, Given the two leading Data, to determine the triangle, when the vertex is posited in any right line, given in position.

These abrupt Outlines of construction, and the remarks, are purposely given as familiar hints to the rising Geometer, for whose further pursuit and improvement, we omit the Demonstrations.

PROBLEM IX.

Given the sum or the difference of a line to which the base has a given ratio, and a line drawn from the vertex, to make a given angle with the base, the difference of the segments of the base, made by the line from the vertex, and the difference of the angles at the base, to determine the Triangle.



CONSTRUCTION.

Make CHK=the first given angle, HK=the given sum or difference, 2CH : HK=the given ratio, 2HP=the difference of the segments, HB=HC, and draw CK, BK; take PQ=PB, making CPQ=the other given angle, and PQR=angle PBK. Then, by Problem I. draw PLR so, that CL : QR =CK : KB, and the triangle is determined.

DEMONSTRATION.

Draw LA parallel to CB, cutting HK at I; also PD parallel to HK, and join A, P.

We shall show, that APL is the Triangle required.

By parallels,

AL : IK=BC : HK,

Or IK is the line to which the base AL, has the given ratio.

Also, the angle PDA=PHK,

Or PD is the line making the given angle with the base,

And KI+DP=KI+IH=KH the given sum.

Again, because of equals and parallels,
AI=IL,
Or AD — DL=2DI=2PH the given difference.
Moreover, by the construction,
CL : QR=CK : KB=CL : BA by parallels,
Then QR=BA ;
And as PQ=PB, and the angle PQR=PBA,
The triangles PQR, PBA, are mutually equal,
Or the angle QPR=BPA :
That is,
The difference of the angles CPL, BPA, or of their equals,
ALP, LAP, is equal to CPQ, the given angle.

◆

This Problem is a general enunciation which we have given to Question 646, Ladies' Diary, proposed by Mr. Wildbore; answered by Mr. Chipchase and Mr. Crakelt: all distinguished Geometers of the day.

◆

From the deductions, we select the following problems, Given in position, a point and three right lines: to draw, from the given point, two right lines, making with one of the right lines, two angles, of which the difference or the sum shall be given, and cutting off, from the two remaining lines, segments having a given ratio to each other.

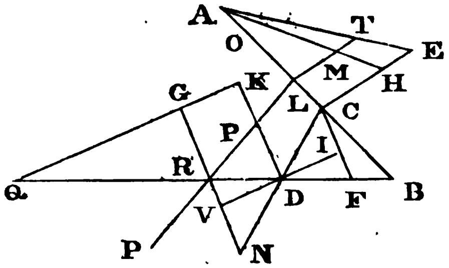
Three right lines and a point, being given in position: to draw, from the given point, two right lines, making, with one of the lines given in position, equal angles, and cutting off, from the two remaining lines, segments, of which the ratio, the sum, or the difference shall be given.

Or, From two given points, to draw two right lines, making equal angles with a right line given in position, and cutting off segments, from two other right lines, given in position, such, that the ratio, sum, or difference, of those segments may be given.

The above are pleasing and useful Problems, and they will amply reward the young Geometer who will take the trouble to give their Analysis and Composition.

PROBLEM X.

To draw, from a given point P, the right line PLR, meeting the right lines BA, BR, given in position, in L, R, so, that drawing LM, RN, to make given angles with AM, DN, two other right lines, given in position, the sum or the difference, of lines to which LM, RN, have given ratios, may be given.



CONSTRUCTION.

From C, the intersection of AB, DN, draw CH, CF, making the given angles with AM, DN, and meeting AM, BD, at H, F: take CH : CE, and CF : FI, equal the given ratios, and join D, I: draw DK parallel to FC, and equal the given Sum, make CE : CA=DK : CO, and let KQ, parallel to ID, meet BD at Q.

Then draw, by Problem I. PLR, so, that AL : QR=CO : DQ.

DEMONSTRATION.

Join A, H: A, E: draw LMT, RNG, parallel to CE, CF, meeting AH, AE, CD, KQ, at M, T, N, G, and let ID meet RN at V.

Because of parallels,

LM : LT=CH : CE,

And

RN : RV=CF : FI;

Or LT, RV, are the lines to which LM, RN, have the given ratios.

Also, VR+RG=VG=DK=the given Sum.

It only remains to be shown, that LT=RG:

By the construction,

DQ : QR=CO : AL,

And by parallels,

DQ : QR=DK : RG;

Therefore,

DK : CO=RG : AL=CE : CA=LT : LA by the construction and parallels;

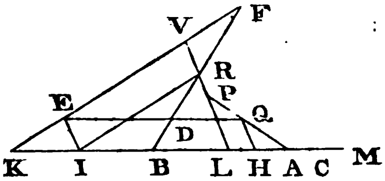
Or LT=RG.

* * *

When the *Difference*, of LT, RV, is supposed to be given, the point K must be taken on the contrary side of D: the other parts of the construction remain as in the preceding case.

This problem is the same as, Through a point P, given in position, to draw a right line PLR, cutting off segments, AL, DR, from two right lines given in position, such, that the Sum or the Difference, of lines to which those segments have given ratios, shall be given.

To which Problem, we may give the following Construction:



Draw PA, take AC=the given difference, PA : AQ, and DB: CM, equal the given ratios; also QE parallel and equal, to MB, and KEF, making BF : BK=DB : CM: then draw PVRL, by Problem I. so, that VE : BR=KF : FB.

Draw RI parallel to FK, join E, I; and let QH be parallel to EI.

By the construction and parallel lines,
BF : FK=BR : VE=BR : RI;
Or VE equal and parallel to RI;
That is,
EI, QH, are parallel to PL:
And therefore, AP : AQ= AL : AH the first given ratio.
Now, as AC =the given difference,
It only remains to show,
That DR : CH= DB : CM:
Obviously, HI=QE=MB, and consequently IB =HM:
Also, by the construction and parallels,
BF : BK=DB : CM=BR : BI,
Then,
BR : BD=BI : CM=HM : CM;
Hence, dividing and alternating,
DR : CH=DB : CM.

The point M must be taken on the contrary side of C, when D lies on the other side of B: and when M falls on the other side of B, QE must be drawn in the opposite direction.

When the construction is for the *sum* of the lines, C must be taken on the contrary side of A.

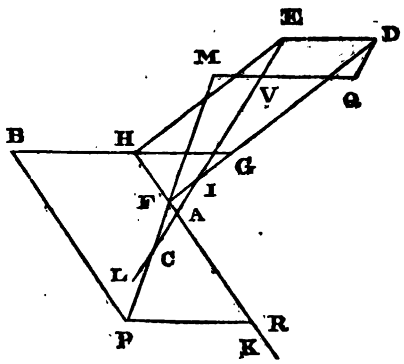
The drawing of KEF, in the above construction, making $BF : BK = DB : CM$, is only drawing, through the given point E, a right line, parallel to a line given in position.

A different solution is given to this problem, in the Book on Analysis, Problem XXVII.

Questions 636, and 766, Ladies' Diary, are particular Cases of this general Proposition.

PROBLEM XI.

From two given points P, Q, to draw the parallel lines PR, QV, meeting two lines AR, AV, given in position, at R, V, and cutting off, from those lines, segments KR, LV, having to each other a given ratio.



CONSTRUCTION.

Draw PB, QD, parallel and equal, to KA, LA, respectively; also DIF, making AF : AI= the given ratio. Then, by Prob. I. draw BHG, so, that AH : DG =AF : FI.

Parallels to BH, through P, Q, are those required.

DEMONSTRATION.

Draw PR, QV, parallel to BH; also HE parallel to FI, and join D, E.

By the construction and parallels,

AF : FI =AH : DG =AH : HE,

Or HE is equal and parallel to GD;

Then, DE is equal and parallel to GH or QV:

That is,

DQ= EV= AL, and therefore, VL =EA.

Also, PB= RH= KA, and consequently, KR= AH:

Hence,

KR : LV =AH : AE =AF : AI,

The given ratio.

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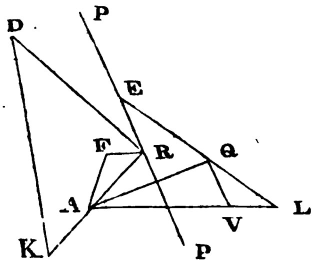
When K or L is posited on the other side of A, the line PB or QD must be drawn in the opposite direction.

When the segments AR, AV, are to have a given ratio; draw PCF, making AF : AC= that ratio: then QVM drawn, by Problem I. so, that AV : PM=AC : CF, determines the position of QV, PR.



OTHERWISE.

Draw QL, KD, making the angle AKD= QLA, and KD: QL= the given ratio: Then drawing, by the *General Problem*, PR, DR, making the angle LER= KDR, it is done.

Draw QV parallel to PR.
By the construction and parallels,
The angle DKR=QLV, and the angle KDR=LER=LQV;
Or KDR, LQV, are equiangular triangles,
Having KR : LV =KD : LQ the given
Ratio.

The Construction for the ratio AR : AV will be much the same as the above.

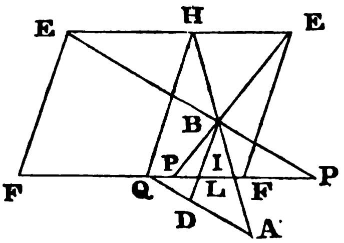
In this case, draw QA, AF, making the angle RAF =LAQ, and AF : AQ =the given ratio: then, by the *General Problem*, draw PR, FR, making equal angles with AQ, AF, it is done.

Draw QV parallel to PR.
Then,
Since the angle RAF= VAQ, and VQA =RFA;
The angle QVA= FRA,
And consequently,
AR : AV= AF : AQ,
The given Ratio, by Construction.

This Problem is the same as the first Case of Question XX. Leeds Correspondent: the preceding furnishes an elegant Determination of that Case.

PROBLEM XII.

The point P and the right lines AB, BD, DA, are given in position : to draw the right line PILQ, meeting the lines given in position in I, L, Q, so, that the segments IL, LQ, shall have a given ratio.



CONSTRUCTION.

Draw PBE, EF, making PB : BE= the given ratio, and EF parallel to BD : Then, by Problem I. draw PQF so, that AQ : EF =AD : DB.

DEMONSTRATION.

Draw QH parallel to DB, and join H, E.
By the construction and parallel lines,
AD : DB= AQ : FE =AQ : QH;
Or FE is equal and parallel to QH :
That is,
EH is parallel to FQ or PQ,
Therefore,
PB : BE =BI : BH =IL : LQ,
The given ratio by the Construction.

OR THUS.

Having drawn PB, and taken PB : BE = the given ratio by Problem XI. draw the parallels PQ, EH, so, that

AQ : AH = AD : AB.

Draw QH.

By the construction,

AQ : AH = AD : AB,

Or QH is parallel to DB :

Wherefore,

PB : BE = BI : BH = IL : LQ,

The given Ratio.

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This Example is taken from Simpson's Geometry: see Problem XXXVII. page 242. fourth Edition.

We hence deduce and determine, the following Problems.

Through a given point, to draw a straight line, constituting, by its intersection with two lines given in position, a Triangle of a given magnitude.

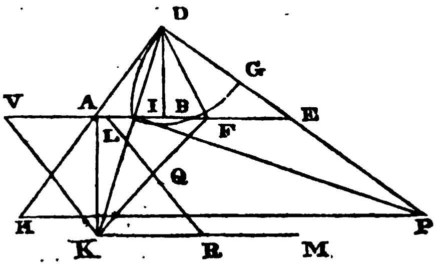
Three right lines and a point, are given in position: to draw a line through the given point, intersecting two of the given lines in two points such, that the Sum or the Difference of the Perpendiculars, from the points of intersection, upon the remaining line, shall be given.

Through a given point, to draw a right line, cutting off segments, from two lines given in position, that shall contain a given Rectangle.

The last Deduction is the same as Problem I. B. II.

PROBLEM XIII.

Given the Chord and the Tangent, to assign the circular Arc.



CONSTRUCTION.

Take FL, and FA= AK, equal the given Chord and Tangent respectively, the angle FAK= AKR a right angle, and join F, K. Then, by Problem I. draw LQR, making FQ :KR= FL : FK, it is determined.

DEMONSTRATION.

To AF, apply KI= KR, also AD, to KI, equal AK: make RM= LF, draw DF; also DB, DE, perpendicular to AF, AD.

We shall show, that E is the centre of the required Arc DI.

Obviously,

The angle AKI =ADI =BDI, and IAD= BDE;

Or the angle DIE=IDE, and therefore, EI =ED:

That is,

The Arc to the centre E and the Radius ED, passes through I, and has its Tangent DA, touching it at D, equal the given Tangent.

Again, by the construction and parallels,

$FQ : FL = KR : KF = KF : KM,$

Or $KF^2 = KR \cdot KM = KI \cdot KM = KI \cdot KD,$

Because of the equal angles KDF, and KFI:

Then, $KD = KM$;

Hence, as KI equal KR,

It follows,

That $ID = RM = LF$, the given Chord.

The Construction might have been referred to Problem XI. thus, having taken FL, FA, AK, as above, and joined F, K, draw the parallels LQ, KV, so, that $FQ : LV = FL : FK$; apply $KI = LV$, and so on, as in the preceding.

The young Analyzer will readily recognise our II. Problem in the above Constructions.

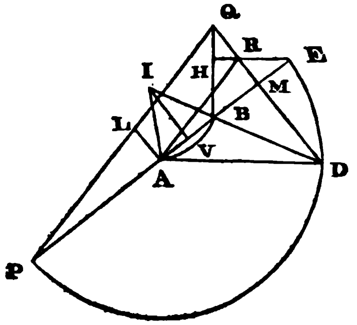
Also, with a little attention, he will, hence, derive solutions to the following Problems.

Given the hypothenuse, and the line bisecting the right angle, to determine the triangle.

To determine the right angled triangle, there are given, one of the sides, and the perpendicular from the right angle, upon the line bisecting the angle opposite to the given side.

From the point A, in the line AD, to draw AE, to E, a point in the perpendicular DE, such, that the right line bisecting the Angle AED, shall touch a given Circle, to the centre D.

In the line DH, perpendicular to DG, a radius of the given circle DGI, is the point A: to draw the Tangent PI, touching the circle at I, so, that joining the points I, A, and drawing PH parallel to IA: the segments AD, AH, shall be equal.



OTHERWISE.

With the Tangent AP, describe the Semicircle PDE, make AB= 2 AV= the Chord, the angle EBH= BEH =half a right angle, and PAL =PVI= a right angle: then, by Problem I. draw PLQ, so, that AL : HQ=EB : BH; it is determined.

DEMONSTRATION.

Draw QM perpendicular to EP, meeting EH at R, and the Semicircle at D; also DI, through B, to VI at I: drawing AI, AD, AR.

We have to show, that the Arc having the Centre I, and Radius IB, is the one required.

As in the preceding, we prove,

That IA=IB,

Or the Arc passes through B, A.

And, as AB, AD, equal the given Chord and Tangent, respectively, we have only to show, that AD touches the Arc at A.

By Construction and similar Triangles,
AL : HQ = EB : BH = RQ : HQ,
Or AL = RQ;
Also, AL, RQ, are parallel,
Then, PQ, AR, are parallel;
Hence, PM : MQ = AM : MR = PM : MB = AM : ME,
And
PM . ME = BM . MA = MD^2;
That is, the Angle DAB = MDB = BIV = AIV:
Consequently,
Angle AIV + IAV = DAB + IAV = IAD, a right Angle;
Or DA touches the Arc BA at A.

In this Case also,
Having described the Semicircle, take AB, AV, and draw EH, BH, as above: then, by Problem XI. let the parallels PQ, AR, be drawn, making HQ = HR: produce QR to the circle at D, and draw DBI.

We have selected this Problem from the Leeds Correspondent, Volume II.

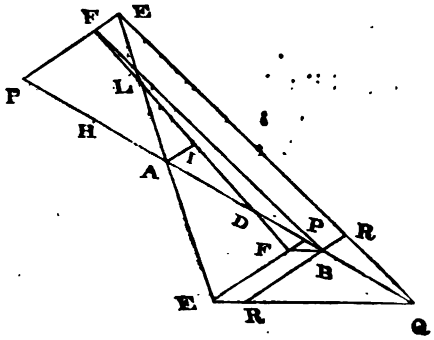
Question 633. Ladies' Diary, and Question 90. Dr. Hutton's Miscellanea Mathematica, as also the VIII Problem, page 12. Lawson's Synopsis, are determinable by the preceding.

We further obtain several simple, but pleasing Problems, as Deductions from the above; the following is selected.

The line AD is perpendicular to the radius DG of the given circle DIG: to draw AIE, making EI = ED.
Figure to the first Construction.

PROBLEM XIV.

Given the points B, D, in the right line AQ; to determine another Point P, in QA produced, such, that the rectangle of the segments PQ, PD, shall have, to the rectangle of the segments PB, PA, a given ratio.



CONSTRUCTION.

Draw, in any position, the Parallels BR, AI, take DA : AI= the given ratio, and let AL, bisecting the angle HAI, meet DI at L: Draw, by Problem I. QRE, making BR : EL= IA : AL; and EP parallel to IA or RB.

DEMONSTRATION.

Let EP intersect DI at F, and draw BF.
By the Construction and Parallels,
$BR : EL = IA : AL = FE : EL$,
Or $BR = FE$;
And BR is parallel to FE,
Therefore, BF is parallel to RE or QE :
Also, the angle $EAI = EAP = AEP$,
Or $EP = PA$;
And $PQ : PE = PB : PF = PQ : PA$;
That is,
$PB . PA = PQ . PF$.
Hence, $PQ . PD : PB . PA = PQ . PD : PQ . PF$
$= PD : PF = AD : AI$,
The given ratio.

The same Construction determines P, when it is proposed to be found between the points B, D.

Problem XI. instead of the I. might here have been used, as our *Auxiliary Problem.*

Thus, take AI, DI, AL, as above: then draw the parallels QE, BF, so, that $LE : LF = LA : LI$; and EF determines P.

The two Cases of this Problem, form the XVII. and XVIII. Problems, B. II. Professor Leslie's Geom. Anal. See also Problems II. and III. pages 36, and 39, Wales' Restitution of Apollonius, on Determ. Sec.

The preceding Construction furnishes us with Solutions to the following Problems.

In a right line, intersecting two circles, given in position and magnitude, to determine a point such, that Tangents to the given Circles, from the point so determined, shall have to each other a given ratio.

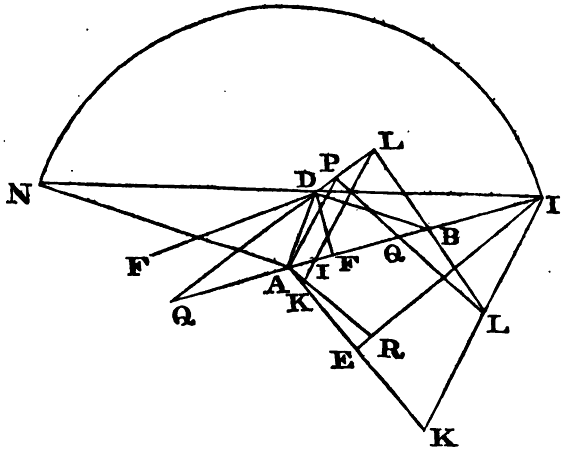
Given the two sides, and the Sum or the Difference, of two Lines to which the segments of the Base, by a Perpendicular from the vertical Angle, have given ratios, to determine the Triangle.

To determine a point P, in the right line passing through the Centers of two given Circles, such, that drawing PQR at right angles to the diameters, to meet the circles at Q, R: the ratio of PQ to PR shall be given.

To add to a given line, or to take from it, another line such, that the Difference between a given Square, and the Square of the compound Line, or the Square of the remaining Line, shall have, to the Difference between another given Square, and the Square of the Line added to, or taken from, the given Line, a given Ratio.

PROBLEM XV.

Given the Sum or the Difference, of the Hypothenuse and one Side, and the segment of the Hypothenuse, adjacent to that side, made by a perpendicular from the right Angle, to determine the Triangle.



CONSTRUCTION.

Take, in any position, AI= IK the given Sum or Difference, AP, parallel to IK, the given Segment, and join A, K. Draw, by Problem I. PQL, making AQ =IL : then LB, parallel to KA, determines AB, the Hypothenuse.

DEMONSTRATION.

Take BF =AP, apply, to the perpendicular FD, BD= BI, and draw DA.

It is to be shown, that ADB is the required Triangle.

Since BD= BI,

We have $AB \pm BD = AB \pm BI = AI$ the given Sum or Difference.
Also, $BF = AP$ the given segment.
It only remains, that we prove ADB a right Angle.
By parallels, construction, and equals,
$QA : AP = QI : IL = IL : BF = BA : IL = IB : BF =$
$BA : BI = BD : BF = BA : BD$;
That is,
BFD, BDA, are equiangular Triangles,
Having the angle BFD=BDA,
Equal a right angle.

* * *

We might have given, to the Construction of this Problem, the form of the XI.

Take AI, IK, AP, as above; draw AK, and IE perpendicular thereto: then, let the Parallels PL, AR, be drawn, making $IL : IR = IA : 2\,IE$: the point L is determined, and hence the whole.

In our References to the XI. Problem, the Demonstrations have been left, as Exercises for the Student.

When, instead of the adjacent, the alternate Segment AF, of the Hypothenuse, is given, the construction becomes extremely easy.

We add the following simple Deductions, as Subjects of further Inquiry.

To take from, or to add to, a given line, another line such, that the rectangle under the remainder, or the Sum, and another given line, shall be equal to the Square of the Segment taken from, or added to, the given line.

Given, two lines, to determine another line such, that adding it to one of the given lines, and taking it from the other; the rectangle under the Sum and the Part added, shall be equal the Square of the Difference.

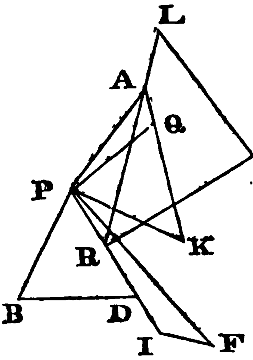
The line FD is perpendicular to the given line AFI: to draw AD, to a point in FD, so, that making ADB a right Angle, IB shall be equal to BD.

From the centre A, of the given circle AIN, to deflect the right lines AN, AD, including a right Angle, meeting the circle at N, and a perpendicular FD, to the Radius AI, at D; so, that the points N, D, I, shall be in the same right line.

To describe a Circle that shall have its center in the line IA, pass through I, and insersect the right line FD, given in position, in D, so, that DA, being drawn, shall be a Tangent to the Circle.

PROBLEM XVI.

The point P and the right lines AK, AL, are given in position: to draw the right lines PQ, PR, including a given Angle, and cutting off segments KQ, LR, from the lines AK, AL, that shall have a given Ratio.



CONSTRUCTION.

Draw PK, PB, BD, LG, making KPB =the given Angle, PB =PK, the angle PBD=RLG=PKA, and PB:LG= the given Ratio: then, by the *General Problem*, drawing PR, GR, making the angle BPR =LGR, it is determined.

DEMONSTRATION.

Draw PQ, making the angle RPQ= BPK: the intersection of PR, BD, being at D.

By the construction,

The angle RPQ= KPB equal the given angle,

Or the Angle KPQ =BPD= LGR;

Also, the Angle PKQ= PBD =RLG,

Therefore,

The angle PQK =PDB= LRG;

That is,

KPQ, BPD, LGR, are equiangular Triangles,

Where

PK : KQ= PB : BD=LG : LR = PB : KQ,

Because of the equals PK, PB :

Whence, alternating,

KQ : LR= PB : LG

Equal the given Ratio, by the Construction.

The same method of Construction, somewhat simplified, may be applied, when the Segments AQ, AR, are to have a given Ratio.

Or, in this Case,

Join P, A; make APF=the given angle, PF = PA, the angle PFI= PAQ: and draw, by Problem I. PRI, making FI : AR =the given Ratio.

Draw PQ, making the angle RPQ=FPA.

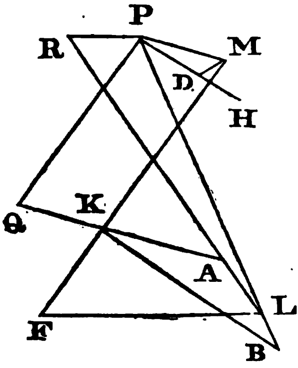
By the construction,

The angle PFI= PAQ, angle IPQ= FPA,

And FP= PA;

Then,

The angle FPI =APQ, angle FIP =AQP,
And the Triangles FIP, AQP, equiangular, and mutually equal,
Or AQ= FI:
That is,
AQ: AR= FI: AR,
The given Ratio, by the Construction.



The general Case otherwise.

Draw PL, KB, PM, PH, making PBK =the given angle, PM parallel to KA, the angle MPH= PLA, and PD: PL =the given ratio: then, by the *General Problem*, drawing KM, DM, so, that the angle BKM =HDM, it is determined.

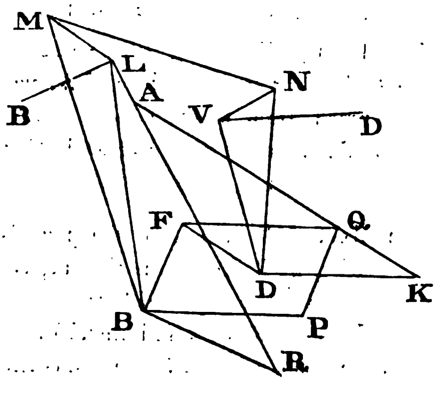
Draw PQ parallel to MK; also, draw the parallels PR, LF, making QPR equal to the given angle.
Because of parallel lines,
The angle RPQ= KFL =KBL,
Or the points K, F, B, L, are in a circle,

Therefore, the angle $BLF = BKF = PDM = LPR$, by the Construction and Parallels;
Hence, PDM, LPR, are equiangular Triangles;
That is,
$PM : LR = PD : PL = KQ : LR$,
Because of the Equals PM, KQ:
Or $KQ : LR$ equal the given Ratio.

This Construction admits of an elegant Application to the particular case, where A, K, L, are coincident.

PROBLEM XVII.

The points P, B, and the right lines AK, AL, are given in position: to draw the right lines, PQ, BR, to include a given angle, and to cut off, from the lines AK, AL, the Segments QK, LR, that shall have a given Ratio.



CONSTRUCTION.

Join P, B, draw KD parallel and equal to PB, and DF parallel to KA: then, by Problem XVI. draw BF, BR, containing the given angle, and making DF : LR= the given Ratio.

DEMONSTRATION.

Draw FQ parallel to DK, and join the Points P and Q.
The opposite Sides of the Quadrilateral KDFQ are parallel,
Therefore, QF =KD, and QK=FD:
Also, by the Construction,
PB=KD,
Then; QF=PB;
The lines QF, PB, are also parallel,
Then PQ is parallel to BF;
Or PQ, BR, contain the given angle.
And, because of Equals
QK: LR=FD : LR
Equal the given Ratio.

* * *

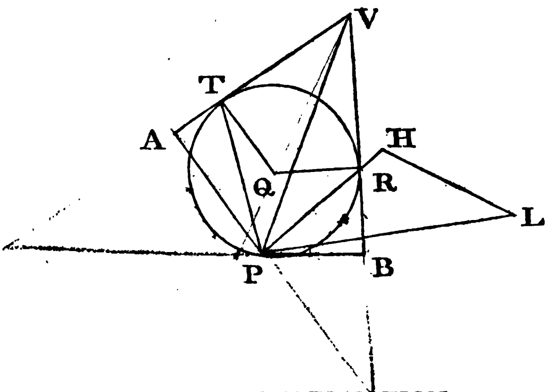
Besides the XVIII. XIX. and XX. Problems, to which we have applied the two preceding ones, as *Auxiliary Problems*, we present, to the ardent Student, the following Deductions.

The right lines BM, DN, and the point L, are given in position: to draw LM so, that making the angle LMN of a given magnitude, the Ratio of BM to DN shall be given.

Given in position, the Points L, V, and the right Lines BL, BM, DV, DN: to draw LM, VN, making equal angles with LB, VD, so, that the right Line passing through the points M, N, shall make a given angle with LM or VN.

PROBLEM XVIII.

To describe, through a given point P, a Circle to touch two right Lines VA, VB, given in position.



CONSTRUCTION.

Let PA, PB, be perpendicular to VA, VB: Then, by Problem XVI. draw PT, PR, including an angle equal half APB, and making VT=VR: the points of contact are T, R.

DEMONSTRATION.

Describe, through P, T, R, the circle PTR, and draw TQ, RQ, to the centre Q.

The Circle passes through the given point P:
Also, the Angle TQR =Double TPR= angle APB,
By the circle and construction:
That is, equal the Supplement of angle TVR:
Or the points T, Q, R, V, are in the Periphery of a Circle,
And therefore, the Angle VRQ+VTQ equal to two right Angles.

Now, VT= VR, and QT =QR,
Or the Angle VRQ =VTQ;
Consequently,
The Angles VRQ, VTQ, are right Angles:
Or the Circle touches VA, VB, at T, R.

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OR, BY PROBLEM I. THUS,

Draw PA, PB, as before; also PV, PL, LH, making the angle VPL= Half APB, PL= PV, and the angle PLH= PVA: Lastly, by Problem I. draw PRH so, that VR=LH.

Describe a Circle through P, to touch VB at R: To demonstrate that the Circle, so described, is the required one, we shall leave as an Exercise for the young Geometer.

The following Deduction is added.
In each of two right lines, given in position, to determine a point, such, that a line passing through those points shall be parallel to another line given in position, and, that a Circle, to contain a given Angle, being described through the same points, shall pass through a given Point.

$$(AV-AT)^2+r^2(AV-AT)^2=VQ^2$$

$$\{BV-(AT+L)\}^2+r^2\{BV-(AT+L)\}^2=VQ^2$$

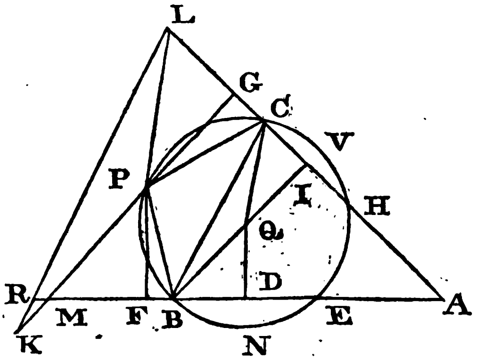
$$AV^2-2AV.AT+\cancel{AT^2}+r^2AV^2-2r^2AV.AT+\cancel{r^2AT^2}$$

$$BV^2-2BV(AT+L)+\cancel{AT^2}+2AT.L+L^2$$

$$r^2BV^2-2BVr^2(AT+L)+\cancel{r^2AT^2}+2r^2AT.L+r^2L^2$$

PROBLEM XIX.

To describe a Circle through a given point P, such, that parts of the Circumference, subtending given Angles, may be cut off, by two right Lines AB, AC, given in position.



CONSTRUCTION.

Draw PF, PM, PG, PL, LR, making F, G, right Angles, FPM, GPL, equal Half the given Angles respectively; and PK=PL: then, by Problem XVI. draw PB, PC, to contain an Angle equal the Supplement of Half KPL, so, that AB : AC=AR : AL.

DEMONSTRATION.

Describe the required Circle through P, B, C, intersecting AB, AC, again, at E, H; from the centre Q, draw the perpendiculars QD, QI: and join Q, B; Q, C; B, C.

By the Circle and Construction,

The Angle BPC equal the Supplement of Half the Angle BQC or of Half KPL;

Or the Angle BQC=KPL:

And therefore, the isoceles Triangles BQC, KPL, are equiangular.

Now, by the Construction,

AB : AC=AR : AL,

Then BC, and RL or KL, are parallel;

Hence, because of equal Angles,

BQ is parallel to KP or MP; CQ to LP; QD to PF; and QI to PG:

Then, the Angle BQD=MPF, and the Angle CQI=LPG;

Whence,

2 Angle BQD=2 MPF, and 2 Angle CQI =2LPG, equal the given Angles, by the Construction:

That is,

The Angles subtended by the Arcs BNE, CVH, are equal the given Angles.

* * *

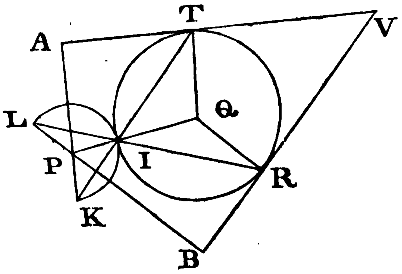
This Example is the same as Problem XLVI. Appendix, Simpson's Algebra.

Among other Deductions, we notice the following.

Two right Lines and a Point, being given in position: to describe, through the given point, a Circle such, that the Segments intercepted, between the point of Concourse of the lines, or between given points in the lines, and the intersections of the Circle, shall have a given Ratio to each other.

PROBLEM XX.

To describe a Circle that shall touch two right lines VA, VB, given in position, and also a Circle, given in position and magnitude.



CONSTRUCTION.

Let the Radii PK, PL, be perpendicular to VA, VB: then, by Problem XVII. draw KT, LR, making the included Angle KIL = Half KPL, and VT = VR: the points of Contact are R, T, I.

DEMONSTRATION.

Through I, T, R, describe a circle, of which, let the Center be Q: draw IP, IQ, TQ, RQ; and let KP, LP, meet VA, VB, at A, B.

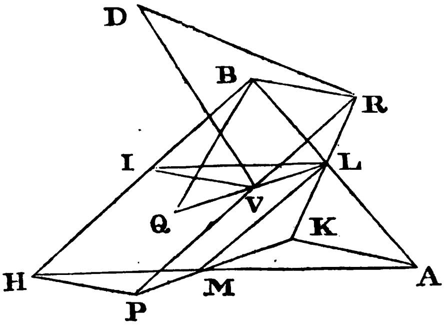
The Angles TQR, 2TIR, 2KIL, KPL, APB, are equal to each other, and each of them equal to the Supplement of the Angle TVR;
Or the points Q, T, V, R, are in the Periphery of a Circle;
Then,
The Angle VTQ+Angle VRQ=Two right Angles;
Also, QT=QR, and VT=VR,
Or the Angle VTQ=VRQ;
That is,
VTQ, VRQ, are right Angles:
Or the Circle touches VA, VB, at T, R.
Moreover, because of parallels and equals,
The Angle PKI=QTI=KIP=TIQ,
Or PIQ is a continued line;
And consequently,
The Circles touch at I.

It seemed superfluous, to prove, that the point I could not fall otherwise, than in the Periphery of the Circle PKL: to the doubtful Geometer and the Tyro, it will furnish a simple Case of the 'Reductio ad absurdum.'

The following Problem presents itself, Two points and three right lines being given in position: to draw a right Line making a given Angle with one of the Lines, and meeting the remaining ones, so, that lines, drawn from the points of intersection, to the two given points, shall either include a given Angle, or have their point of Concourse in another right Line, given in position.

PROBLEM XXI.

Given in position, the points P, Q, R, and the right Lines AK, AL: to draw the parallels PK, QL, meeting the Lines given in position, at K, L, so, that the right Line passing through the points K, L, shall also pass through R.



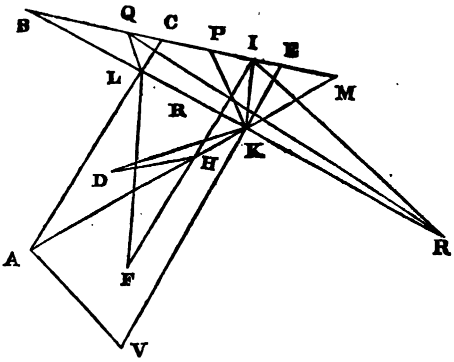
CONSTRUCTION.

Draw RB parallel to AK, join P, R; Q, B: make the Angle PRD=ABQ, and BQ : RD = AB : PR. Then, by the *General Problem*, draw DV, QV, so, that the Angle RDV = BQV: it is done.

DEMONSTRATION.

Let QV meet AB at L, draw PK parallel to QV or QL: Also, VI, PH, parallel to RB, meeting BH, drawn parallel to RP, at I, H: draw IL, HA, the latter meeting PK at M; and join M, L; K, L; R, L.

By the Construction,
The Triangles RDV, BQL, are equiangular,
Or BQ : RD = BL : RV = BA : RP,
Hence,
BL : BA = RV : RP = BI : BH,
And IL, HA, consequently, parallel:
Now, HP, HM, PM, are respectively parallel, to IV, IL, VL;
And HP = IV:
Therefore,
MH = LI, and ML parallel to HB:
Then,
BL : LA = HM : MA = HP : AK = BR : AK;
Or KLR is a continued Line,
That is,
KL passes through R.



OTHERWISE.

Let PQ meet AL at C, take CI= QP, draw IH, RQ, HD, making IH parallel to CA, the Angle AHD =PQR, and RQ : QC= HA : HD: then, by the *General Problem*, drawing RK, DK, so, that the Angle QRK =HKD, it is determined.

Draw KP, LQ: also KE parallel to HI or LC, and produce RL to PQ at B.

The Triangles RQB, KHD, are obviously similar, having
RQ : QB =KH : HD,
Or RQ . HD= QB . KH= QC . HA, by Construction ;
Then,
QB : QC =HA : HK= IC : IE =QP : IE,
By parallels and equals;
Hence, alternating and compounding,
BE : BC =BP : BQ= BK : BL,
Because of the parallels KE, LC:

Or PK, QL, are parallel,
And KL passes through R.

◆

The point M, the intersection of PQ with the other line AK given in position, might have been used, in the above, instead of the point C.

Hence, when PQ is parallel to either of the lines AL, AK, given in position; by its intersection with the remaining line, we are enabled to apply the preceding Construction.

Or in this particular Case, we may assign the position of RKL, as follows.

Let the line IF, passing through the given Points I, F, and meeting AK at H, be parallel to AL, one of the lines AK, AL, given in position: to draw the parallels IK, FL, so, that KL shall pass through R: or, to assign such a position to RKL, that IK and FL shall be parallel.

Draw IR, AV, making the Angle HAV = HIR, and IR : IF = AH : AV: Also, by the *General Problem*, draw RK, VK, so, that the Angle IRK = AKV.

The Demonstration is omitted: it may be advantageous to the advancing Student, to demonstrate the Parallelism of the lines IK, FL.

A simple and elegant solution, to the second Case, of Question XX. Leeds Correspondent, is hence obtained.

Determine the following, by applying the XI. as an Auxiliary Problem.

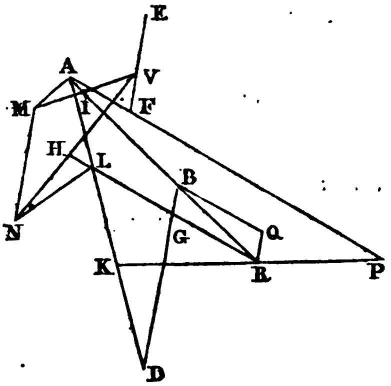
To construct the Quadrilateral, when one side is given in length and position, the two sides adjacent to the given one, are parallel, the remaining side passes through a given point, and the angular points, at its extremities, are posited on two right-lines, given in position.

PROBLEM XXII.

The point R, and the lines DK, DB, are given in position: to draw the lines RL, RA, meeting the Lines given in position, at L, A; and G, B: so, that the ratio of KL to LA shall be given, and the Segment GB, of a given length.

CONSTRUCTION.

Join K, R; make KR : RP=the given Ratio, and RQ, parallel to DB, equal the given Line: then, by Problem XXI. draw the parallels PA, QB, so, that AB shall pass through the point R.



DEMONSTRATION.

Draw RGL parallel to PA or QB.
Because of parallels,
KL : LA = KR : RP,
The given ratio;
Also,
The Quadrilateral QBGR, is a Parallelogram,
Or GB = RQ;
That is,
GB
Is equal the given Line.

The 204th Question, Leybourn's Repository, is a particular Case of this Problem. See page 441. Old Series. We give the following deduced Problem, as a subject of further Inquiry.

The point R, and the lines AL, MN, EF,

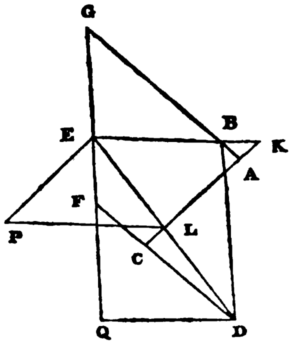
are given in position, of which, MN, EF, are parallel: to inflect MV, NV, to a point V, in EF, such, that making MI : IV=NH : HV equal a given ratio, and drawing RA, RL, through I, H; the right lines passing through the points M, A; and N, L; shall be parallel.

PROBLEM XXIII.

Given in position, the point E, and the right lines AB, CD, EF, AC: whereof AB, CD, are parallel: to determine the position of BD, parallel to EF, so, that drawing EB, ED, meeting AC at K, L; the intercepted Segment KL shall be given.

CONSTRUCTION.

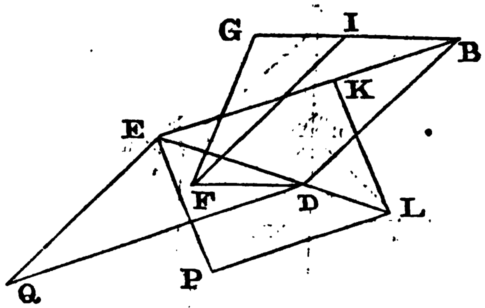
Let EF meet AC, AB, at F, G: make EQ=FG, and EP, parallel to AC, equal the given Line: Then, by Problem XXI. draw the parallels PL, QD, so, that DL shall pass through E.



DEMONSTRATION.

Draw DB parallel to FG, and let EB meet CA at K.
The Quadrilateral BDFG, is a Parallelogram,
Or
DB=FG=EQ,
By Construction;
Then EBDQ also, is a Parallelogram,
Or EB, EK, parallel to QD, PL:
Also EP is parallel to KL,
Hence,
EKLP also, is a Parallelogram,
Or KL=EP:
That is,
KL
Is equal the given Line.

A more general and pleasing Form may be given to this Problem, by supposing the point E to be any how posited, and the position of BD to be assigned, making a given Angle with FG, instead of being parallel to it.



IN THIS CASE,

Make GFI equal the given Angle, EQ parallel and equal, to IF; EP equal to KL, the given Segment, and parallel to the line on which that Segment is to be intercepted: from P, Q, draw, by Problem XXI. the parallels PL, QD, meeting the lines KL, FD, so, that LD shall pass through the point E.

The rest may be supplied by the Reader.

This Generalization, and the original Problem, are only particular cases of the following Enunciation.

Given in position, four right lines and a point: to draw, from the given point, two right lines that shall intercept, on one of the right lines given in position, a Segment of a given length, and meet two of the remaining lines given in position, so, that a line passing through the points of intersection, shall be parallel to the fourth line given in position.

See the Remarks at the end of Problem XXVI. B. I.

The *Maximum* Case of this Problem was proposed and answered, Fluxionally, in the Mathematical Companion: a GEOMETRICAL Consideration, of the same Problem, may be seen in No. I. Vol. IV. of the Leeds Correspondent.

We, hence, make the following obvious Deductions.

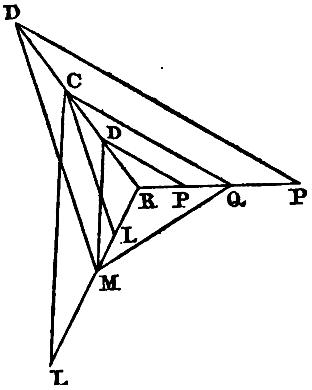
Given, the Diagonals, the Angle they make with each other, and the position of one of them, to determine the Trapezium, when either pair, of the opposite Sides, has its point of concourse given in position.

Given, the Diagonals, their point of intersection, the Angle they make with each other, and the position of one of them, to constitute the Trapezium, when either pair, of the opposite Sides produced, intersects each other in a right line given in position.

PROBLEM XXIV.

The right lines CQ, CR, and the points M, P, are given in position: to draw the lines MQ,

MR, including a given Angle, and meeting the lines given in position, at Q, R, so, that the Line passing through the points Q, R, shall pass also through P.



CONSTRUCTION.

Draw PD parallel to CQ, join D, M ; and let CL be parallel to DM : then, by Problem XVI. draw MQ, ML, including the given Angle, and making CQ : CL=DP : DM.

DEMONSTRATION.

Let ML meet CR at R, and draw PQ, QR.

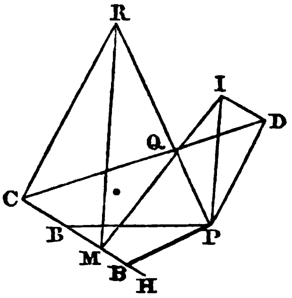
By the Construction,

QMR=the given Angle ;

Also,

CQ : DP=CL : DM=CR : DR,

Because of the Parallels CL, DM;
By Alternation,
RC : CQ = RD : DP,
Hence, as CQ, DP, are parallel,
It follows, that RQP is a continued line;
That is,
RQ passes through P.



OTHERWISE.

Draw CM, PB, PD, DI, making PBH equal the given Angle, and PD, DI, parallel to CR, CM: then, by the *General Problem*, draw MI, PI, so, that the Angle CMI = BPI.

Let the intersection of MI, CD, be at Q.
Make QMR equal the given Angle, and draw QR, QP.
By the Construction, the Angle
CMI = BPI,
Or the Angles
BPI and BMI,
And consequently, the Angles
PBM and PIM,
Are, together,
Equal to two right Angles;

Or the Angle
PIM=PBH=RMI,
And PI, RM, are, therefore, parallel:
Hence, because of the similar Triangles
RCM, PDI,
MR : IP=CM : DI=QM : QI,
That is, RQP is a continued Line,
Or
RQ passes through P.

◆

The Line PD, in these Constructions, may be drawn parallel to either of the Lines CR, CQ, given in position.
This Problem has been proposed in the Mathematical Companion, Question 148, page 240.

The following Problems appear, among other Deductions.

One of the angular points of a Triangle is given in position, also the magnitude of that Angle: to determine the Triangle, when the other Angles are posited upon two right Lines given in position, and the Side, opposite the given Angle, passes through a given point.

To describe a Triangle, similar to a given Triangle, having one of its angular points given in position, and the remaining Angles posited upon two right Lines, given in position.

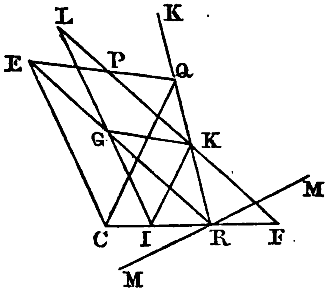
To determine a Point, in a right Line given in position, such, that drawing, thence, a Line to a given point, also a Line making a given Angle with that drawn to the given Point, and a Line parallel to another, given in position: the two latter shall cut off, from two other Lines given in position, Segments, from given Points therein, of which, the Ratio, Sum, or Difference shall be given.

PROBLEM XXV.

The points P, M, K, and the right Lines CQ, CR, are given in position: to draw the Lines PQ, MR, containing a given Angle, and meeting CQ, CR, at Q, R, so, that the Line joining the Points Q, R, shall pass through K.

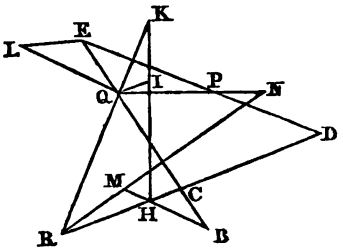
CONSTRUCTION.

Draw PK, meeting CR at F, KI parallel to QC, make IF: FL = IC : PK, and join I, L. Then, by Problem XVII. draw KG, MR, including the given Angle, and making IG : IR = IL : IF.



DEMONSTRATION.

Draw RG; also PQ parallel to GK, meeting CQ,
RG, at Q, E: and draw CE, KQ, KR.
Now, PQ is parallel to GK,
And GK, MR, include the given Angle,
Therefore, PQ, MR, include the given Angle:
Also, by the Construction,
IG : IR = IL : IF,
Or RG is parallel to FL,
And therefore, KPEG is, a parallelogram;
Then GE = KP.
Again, by Parallels, Construction, and Equals,
IR : RG = IF : FL = IC : PK = IC : GE,
Or, CE, IG, are parallel;
Hence, IGK, CEQ, are equiangular Triangles,
Where IG : CE = GK : EQ = RG : RE,
Because of parallels;
The Points
R, Q, K,
Are, therefore, in a right Line,
That is,
QR
Passes through K.



OTHERWISE.

Draw MB, PD, making MBC= PDC= the given Angle, meeting CR, CQ, at H, D, B, E: also, draw KH, EL, making Angle CEL= KHR, and HK : HM= PE : EL. Then, by the *General Problem*, draw KQ, LQ, so, that Angle
HKQ= EQL.

Join P, Q; draw MR, making the given Angle with PQ, and meeting it at N: draw QI parallel to RH, and join Q, R.
By the Construction,
The Angles HBQ, HDP, MNQ, are equal,
Or the Angle
DHB= DEB= RHM,
And the Angle
NRD= NPD= QPE;
Therefore,
The Triangles RHM, EPQ, are similar,
Having
RH : HM= PE : EQ,
Or HM . PE= RH . EQ= HK . EL, by the Construction;
Hence, RH : HK= LE : EQ= QI : IK, because
Of the equiangular Triangles
LQE, QKI;

That is,
KQR
is a continued Line,
Or
QR passes through K.

The following pleasing Deductions are added, for the consideration of the Geometer.

Given one of the Angles to determine the Triangle, when the sides pass through three given Points, and the remaining Angles are posited upon two right Lines given in position.

To determine the Trapezium: one of the sides is given in length and position, the side opposite the given one, passes through a given point, the Angles, at its extremities, are posited upon two right Lines, given in position, and the remaining sides include a given Angle.

Two Points and two right Lines, are given in position: to draw, from one of the given points, two Lines, including a given Angle, meeting the Lines given in position, so, that right Lines drawn from the Intersections, to the other point given in position, shall include a given Angle.

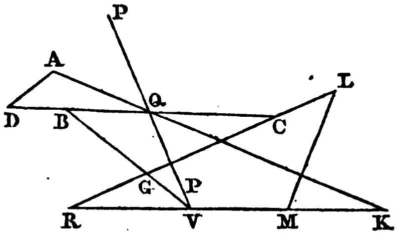
Three Points and two right Lines being given in position: to draw, from one of the given points, two lines, containing a given Angle, and meeting the Lines given in position, so, that Lines, from the points of intersection, to the two remaining points given in position, shall be parallel to each other.

PROBLEM XXVI.

The right Lines CQ, CR, BG, and the points M, P, are given in position: to draw the lines MV, PV, having their point of concourse V, in the line BG, and cutting off, from the lines CQ, CR, the segments DQ, LR, having a given Ratio.

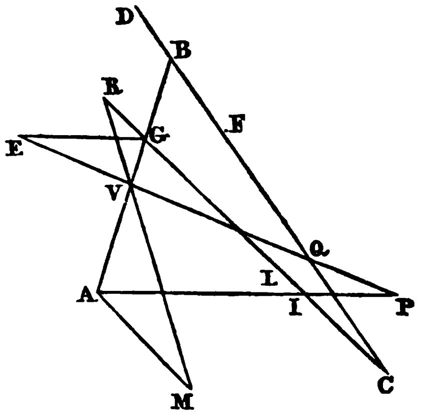
CONSTRUCTION.

Draw LM, DA, making the Angle CDA=GLM, and DA : LM equal the given Ratio: then, by the XXV. Prob. draw MV, AQ, including an Angle=DCG, so, that VQ shall pass through P.



DEMONSTRATION.

Let MV meet CG, AQ, at R, K.
By the Construction, VR, VQ, pass through the points
M, P,
And intersect at V, a point in
BG:
Also, the Angle
QLR=QCR,
Or the points
C, K, Q, R,
Are in the Periphery of a Circle, and therefore, the Angle
CQK=CRK=AQD=MRL;
Hence, as the Angle
QDA=RLM,
The Triangles QDA, RLM, are equiangular,
Having
DQ : LR = DA : LM,
The given Ratio.

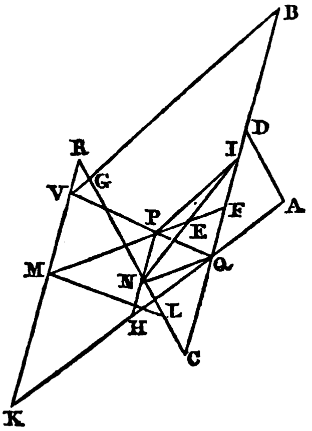


OTHERWISE.

Draw MA, PA, GE, making MA, GE, parallel to CG, PA; and DF : GL=IA : AM equal the given Ratio: then, by Problem I. draw PQE, so, that FQ : GE=IA : AP.

Through V, the Intersection of PQ, BG, draw MVR, to CG at R.

By the Construction,
GE : AP = FQ : IA = VG : VA = GR : AM,
Because of parallel lines;
Then alternating,
FQ : GR = IA : AM = FD : GL = DQ : LR,
The given Ratio.



OR THUS.

Draw MP meeting BC at F; also, ML, PI, DA, IE, making PI parallel to GB, the Angle CDA=GLM, the Ratio IF : FE= BI : PM, and DA : LM=.the given Ratio: then, by Problem XVII. draw PN, AQ, including the Angle PHQ=BCG so, that IN : IQ equal IE : IF.

Draw QN, and let AQ meet RM at K.
By the Construction,
IN : IQ=IE : IF,
Or
QN is parallel to PM,
Then,
IF : FE =IQ : QN=BI : PM,
Or alternating, QI : IB= QN : PM =QP : PV, by parallels;

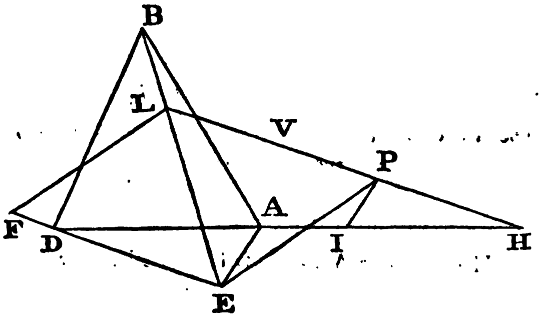
Whence,
NQ : QP=MP : PV,
Or
PN, KR,
also, are parallel:
Hence,
The Angle RKQ= QHP= QCR, by Construction;
And therefore, the Angle
KQC=KRC=AQD:
That is,
The Triangles ADQ, MLR, are equiangular,
Having
DQ : LR=DA : LM,
The given Ratio.

◆

This Problem has been elegantly solved by Professor Wallace, and Mr. Butterworth, Math. Repo. Vol. I. pa. 43. New Series.

Problem XXVI. leads to an easy Determination of that which we enunciated at the Close of the XXIII., which is,

'To draw, from a given Point E, two right Lines EB, ED, to intercept, on LF, a line given in position, a Segment LF, given in Length, and meet two other Lines AB, AD, also given in position, at B, D, so, that BD, the Line connecting the Points B, D, shall be parallel to another Line given in position: or, which is the same, that the ratio AB : AD shall be given.'



Join A, E; draw EP, PI, parallel to FL, AE, making EP=FL, the given Segment, and PV : AE=the given Ratio. Then, from P, E, draw, by Problem XXVI. PL, EL, intersecting at L, in the Line FL, and meeting IA, BA, at the points H, B, so, that IH : AB=PI : PV.

Draw ED parallel to PL, meeting AD, LF, at D, F.

The Quadrilateral EPLF has its opposite Sides parallel to each other, it is, therefore, a Parallelogram;
Or
LF=PE,
The given Segment:
Also, because of parallel Lines,
IH : IP=AD : AE,
And, by the Construction,
IH : IP=AB : PV,
Consequently,
AB : AD=PV : AE,
The given Ratio.

The following MORE GENERAL FORMS may be determined in the same Way.

To draw, from a given Point, two Lines, intercepting, on a Line given in position, a Segment of a given Length, and cutting off, from two other Lines given in position, between the points of intersection and given points in those Lines, two Segments, having a given Ratio to each other.

Given, in position, four right Lines and two points: to draw, from one of the given points, two Lines, meeting two of the Lines given in position, at P, Q, and cutting, therefrom, Segments having a given Ratio, Sum or Difference, so, that Lines drawn from the remaining given point, to the points P, Q, shall cut off, from the two remaining Lines given in position, Segments, of which the Ratio, Sum or Difference, shall be given.

Problem XXII. is only a particular Case, of the first of the above Deductions.

As a *closing* Subject of Inquiry, for the Geometrician, we present the following deduced Problem.

Four right Lines and three Points, are given in position: to draw, through one of the given Points, a right Line, meeting two of the Lines given in position, so, that Lines drawn from the Points of Intersection, to the two remaining Points, given in position, shall make equal Angles with the two remaining Lines, given in position.

Problem VIII. B. II. is a particular Case of this Deduction.

◆

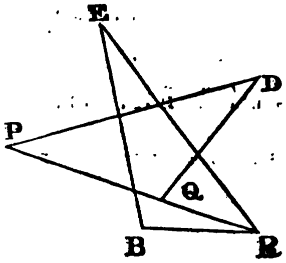
Having given, in CONSTRUCTION, a sufficient Number and Variety of Problems, to show the Extent and Facility of Application, of our GENERAL PROBLEM and its AUXILIARIES ; We now proceed to BOOK II. which contains a Series of appropriate and diversified Sketches of ANALYSIS.

BOOK II.

ANALYSIS.

PROBLEM I.

The right Lines DQ, ER, and the point P, are given in position: to draw the Line PQR, meeting DQ, ER, at Q, R, so, that the Rectangle of the Segments DQ, ER, shall be given.



ANALYSIS.

Suppose PQR drawn as required.

Draw PD, EB, RB, making the Angle REB=PDQ, and the Angle ERB=DPQ.

Now,

RE is given in position; E is a given Point; and REB equal PDQ, is a given Angle;

Or EB is given in position.

Also,
The Triangles PDQ, REB, are equiangular,
Having
PD : DQ=RE : EB,
Or
PD . EB=DQ . ER,
is a given Rectangle, and PD a given Line;
Then,
EB is given in Length, as well as in Position:
That is,
B is a given Point.

The Points P, B, and the right Lines PD, ER, then, are given in position: and, we have to draw PR, BR, to R, a point in ER, such, that the Angles DPR, ERB, may be equal.

THIS IS THE GENERAL PROBLEM.

COMPOSITION.

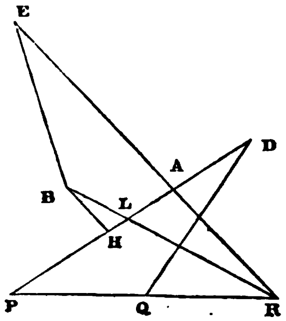
Draw PD, EB, making the Angle REB=PDQ, and PD . EB equal the given Rectangle.

Then, by the *General Problem*, draw PR, BR so, that the Angle DPR= ERB.

DEMONSTRATION.

By the Composition, the Angle
ERB= DPR, and PDQ= REB;
Or
The Triangles REB, PDQ, are equiangular,

Having
RE : EB=PD : DQ ;
That is,
DQ . ER=PD : EB,
The given Rectangle.



OTHERWISE.

Suppose the position of PQR to be determined.

Draw PD, meeting ER at A ; also EB, RB, as in the preceding ; and BH parallel to ER : the intersection of PD, RB, being at L.

The line ER and the point E, are given in position,
And REB equal PDQ, is a given Angle ;
Or EB is given in position.
Also,
Because of the similar Triangles
PDQ, REB,
QD : DP=BE: ER,
Or
DP . EB =DQ . ER,
is a given Rectangle, and DP a given Line;

That is,
EB is given:
Consequently, B, and, thence, the Line BH, are given:
Now,
The Triangles RAL, PAR, are equiangular,
Having RA : AL=PA : AR : BH : HL, by Parallels;
Hence,
AR : HL= PA : BH,
is a given Ratio.

The point B, and the lines AR, HL, therefore, are given in position: and the line BLR is to be drawn, cutting off, from the Lines given in position, the Segments AR, HL, having a given Ratio.

THIS IS PROBLEM I. B. I.

COMPOSITION.

Draw PD, meeting ER at A, also EB, BH, making the Angle REB=PDQ, the Rectangle PD.EB=the given one, and BH parallel to ER.
Then, by Problem I. draw BL, so, that HL : AR=BH : PA.

DEMONSTRATION.

By the Composition and Parallels,
LH : HB=RA : AP=LA : AR,
Or the Triangles PAR, RAL, are equiangular,
Having the Angle
APR=ARL=ERB:

Also, by the Composition,
Angle PDQ = REB, or
The triangles PDQ, REB, also, equiangular,
Having
DP : DQ = ER : EB ;
That is,
DQ . ER = PD . EB,
The given Rectangle.

The above is the same as the Problem of the Ancient Geometricians, 'DE SECTIONE SPATII.' See pages 264. and 435.
LESLIE'S ELEMENTS, 2nd Edition.

PROBLEM II.

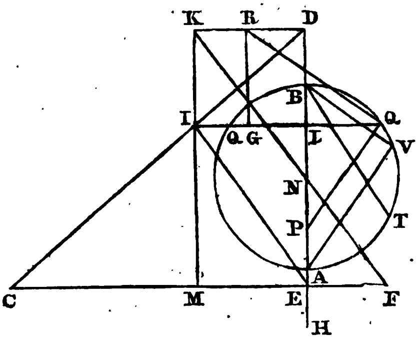
The Line DR is perpendicular to the diameter AB of a given Circle, and P is a given point, any how posited in DA : to determine a point Q, in the periphery of the circle, such, that drawing PQ ; and QR, making a given Angle with DR, the ratio of PQ to QR, may be given.

ANALYSIS.

Suppose Q determined :
Also PQ, QR, drawn,
and let the Center of the Circle be at
N.
Make AH : HB = AP : PB, bisect PH at E, and draw
QL parallel to DR.

The Ratios PQ : QR, and LD : QR, are given,
Therefore, PQ : LD, and $PQ^2 : LD^2$, are given Ratios :
Now,
$PQ^2 = EL . 2PN$,
Or
$EL . 2PN : LD^2$
is a given Ratio.

In QL, take LI = 2PN, and let DI, and IM, a parallel to BA, meet EC, a parallel to QI ; and DR, at C, M, K.



The points H, E, are given,
And EC is parallel to a Line given in position,
Or EC is given in position :
Also,
Because of Parallels,
DK = LI = 2PN,
a given Line,
Or K is a given point,
And KM parallel to DE :
The Line KM, and the point M,
are, therefore, given in position.

Moreover,

$$DL : LI = IM : MC = DL : 2PN = LE : MC,$$

Then,

$$EL . 2PN = DL . MC ;$$

And consequently,

$$DL . MC : DL^2 = MC : DL = MC : KI,$$

is a given Ratio.

THAT IS,

We have to draw, from the given point D, the Line DIC, meeting the Lines KM, EM, given in position, at I, C, so, that the Segments KI, MC, shall have a given Ratio.

THIS IS PROBLEM I. B. I.

COMPOSITION.

Having taken the Points H, E, K, and drawn EC, KM, agreeably to the Analysis; make BAV equal the given Angle, TB : BA = the given Ratio, and join B, V.
Then, by Problem I. B. I. draw DIC, so, that

$$KI : MC = BV^2 : BT^2.$$

DEMONSTRATION.

Draw ILQ perpendicular to AB; join P, Q; and draw QR, RG, parallel to VB, DB.

By the Composition, Equals, and Parallels,

$$KI : MC = BV^2 : BT^2 = DL : MC = DL^2 : DL. MC = DL^2 : EL . LI = DL^2 : EL . 2PN = DL^2 : PQ^2$$

Or

$PQ : BT = DL : BV.$

Now, the Angle

$QRG = ABV,$

And G, V, are right Angles;

Therefore,

The Angle $RQG = BAV = QRD$ equal the given Angle,

And the Triangles QRG, ABV, equiangular,

Or

$DL : BV = QR : BA = PQ : BT;$

That is,

$PQ : QR = TB : BA,$

The given Ratio.

THE COMPOSITION OTHERWISE:

Take the Points H, E, and draw EC, AV, BV, BT, as in the preceding; make $EC = ED$, draw CD, and take $EF : 2PN$ equal $BV^2 : BT^2$.

Then by Problem XI. draw the Parallels EI, FK, so, that

$DI : DK = DC : CE.$

The Demonstration, and the additional, essential Lines, are left, as Exercises.

From the preceding, a Solution is obtained, to the following Problem.

The Diameter AB, of a given Circle, a point P, in that diameter, and the right Line MR, are given in position; to determine the point R, in MR, such, that PQ, having a given Ratio to MR, being applied to the circle, the right Line passing through Q, R, shall be at right Angles to AB.

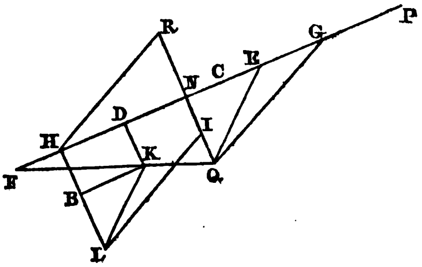
We have given, to the above, and Problem II., a more general Form, in the VI. Problem, and its Deduction.

PROBLEM III.

In the right Line PF are given, the points G, E, C: to determine the Points N, D, such, that the Ratio of ND to DF, as also the Ratio of the Square of NG to the Rectangle ED . PC, may be given.

ANALYSIS.

Suppose the Points N, D, determined as required.
Take, in any position, GQ=PC, draw FQ, NQ; also DK, parallel to NQ.
The points Q, F, and therefore QF, are given;
Also,
QK : KF=ND : DF,
is a given Ratio,
Or K is a given Point.
Draw EQ; and KL parallel and equal, to EQ: also LH; KB; LI, HR; parallel to QN; DH; QG.
The point K is given,



And KL is equal and parallel to EQ, a given Line:
Then,
L is a given point,
And consequently,
LI, parallel to QG,
is given in position.
Also, because of Parallels,
The Triangles EQN, KLB, are similar,
And, as
$KL = EQ$,
They are also mutually equal,
Having
$KB = EN = DH$;
That is,
$ED = NH$.
Now,
Because of parallel lines and equals,
$NG : GQ = NH : HR = NG : PC = ED : LI = NG^2 : NG . PC$
$= ED . PC : LI . PC$;
Or alternating,
$NG^2 : ED . PC = NG . PC : LI . PC = NG : LI$,
A given Ratio.

Whence, The point Q and the lines GN, LI, are given in position: and our Problem is, to draw QIN, so, that the Segments GN, LI, shall have a given Ratio.

WHICH IS PROBLEM I. B. I.

COMPOSITION.

Draw GQ, EQ, FQ, to any point Q, making GQ =PC, and QK : KF equal the first given Ratio; take KL, LI, parallel to EQ, QG, and KL=EQ.

Then, by Problem I. B. I. draw QIN, so, that GN : LI equal the other given Ratio.

* * *

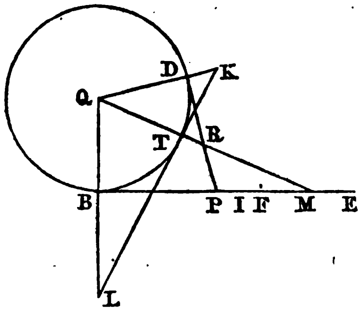
After giving the ANALYSIS and COMPOSITION of the Problem, we now deem it superfluous to add the DEMONSTRATION: if desirable, the Improvement, to be derived from the Completion, will abundantly compensate the Labour of the young and emulous Geometrician; to whom we recommend the Pursuit.

The following Deduction has been thought too interesting to be omitted.

The line DR, and the Parallels PQ, KL, are given in position: to draw PR, QR, to intersect at a point R, in the Line DR, so, that meeting KL at the points M, N; the Sum or the Difference of KN, and a mean proportional between KL, KM, shall be equal to a given Line.

PROBLEM IV. ✓

An Arc, of a given Circle, being given in magnitude: to assign the position of a Tangent to the Arc, so, that the part of the Tangent intercepted by Radii, through the extremities of the given Arc, shall be of a given Length.



ANALYSIS.

Let the required Tangent KL touch the given circular Arc BD at T.

Draw, touching the circle, at B, D, the Tangents BP, DP, intersecting the radius QT produced, at M and R.

The lines BM, PR, are given in position.

In BP,

Take BE=KL, BI=IE, and IF=IP.

Then BP, also BE, and therefore BI; are given;

Hence,

PI, IF, PF, and BF,

are given Lines,

Or F is a given point, in BM.

Now, obviously,

$BP = DP$, $TK = DR$, $TL = BM$, and $TK + TL = KL = DR + BM = 2BP + PF$.

Also,

$DR = DP - PR = BP - PR$,

And

$BM = BP + PM$:

That is,

$PR = PM - PF = FM$.

WHENCE,

The point Q, and the Lines PR, FM, are given in position, and P, F, given points in those lines: to draw QRM, cutting off, from PR, FM, the equal Segments PR, FM.

This is a PARTICULAR CASE OF PROBLEM I. B. I.

COMPOSITION.

To the given Arc BD, draw the Tangents BP, DP; in BP, take BI equal Half the given Line, and $IF = IP$.

Then, by Problem I. B. I. draw QRM, from the centre Q, making $PR = FM$: the point of contact T, of the required Tangent, is determined by QM.

The above Problem is the 243. Question, MATHEMATICAL REPOSITORY, New Series.

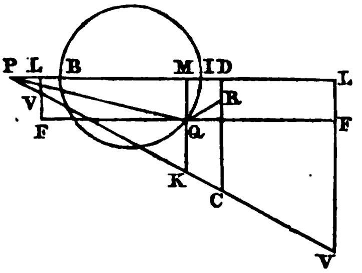
The foregoing Analysis is applicable to Problems under the following Forms of Enunciation.

To divide a given circular Arc into two Parts, such, that the Sum or the Difference, of two Lines, to which the Tangents of those parts have given Ratios, shall be equal a given Line.

Given the vertical Angle, the Length of a Line, from that Angle, making a given Angle with the Base, and the Sum or the Difference, of two Lines, to which the Segments of the Base, by the Line from the vertex, have given Ratios, to determine the Triangle.

PROBLEM V.

The Line DR is perpendicular to BI, the Diameter of a given Circle, and P is a point posited in DB: to determine a point Q, in the Periphery, such, that drawing QR, making a given Angle with DR, and joining Q, P; the Sum or the Difference, of PQ, QR, shall be equal a given Line.



ANALYSIS.

From Q, which we suppose determined, draw QM, parallel to RD, meeting PC, drawn to make PCD equal QRC, the given Angle, at K:

Take KV =QP, and let LF, a parallel to MK or DC, through V, meet DP, and QF, a parallel to DP, at L, F.

The Point P and the Line DC, are given in position,
and the Angle DCP equal a given Angle;
Or PC is given in position,
and C a given point.
Also,
Because of equal Angles and Parallels,
KC= QR;
Hence, as
KV= QP,
Obviously, CV= KV±KC =PQ±QR equal
a given Line,
That is,
V is a given point:

And LF, passing through V, being parallel to DC, or perpendicular to PD, is therefore given in position.

Moreover,

Because of equals and parallel lines,

VK : LM=QP : QF =PC : PD

equal a given Ratio.

CONSEQUENTLY,

The Line LF is perpendicular to the Diameter BI of a given Circle, and P is a given point in BI: to determine Q, in the Periphery, such, that drawing QF, making a right Angle with LF, and joining Q, P: the Ratio PQ : QF shall be given.

THIS IS PROBLEM II. B. II.

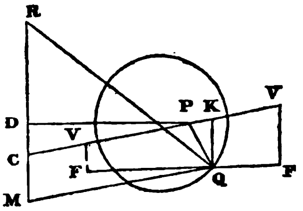
COMPOSITION.

Make PCD equal the given Angle, CV equal the given Line, and draw VL parallel to CD.

Then, by Problem II. B. II. determine Q, such, that drawing QF at right Angles to LV, and joining Q, P; the Ratio QP : QF may be equal PC : PD.

* * *

We may analyse and determine the Problem in nearly the same way, when the Sum or the Difference, of Lines, to which PQ, QR, have given Ratios, is equal a given Line.



Let QM be the Line to which QR has the given Ratio, and
draw PC parallel to QM.
The Angle QRM, and the Ratio QR : QM are given,
Or QMR= PCR, is
a given Angle :
That is,
as the point P, and the line CD, are given in position,
PC and the point C, are given in position.
Draw QK parallel to CD, let KV be the Line to which PQ
has the given Ratio, and draw VF, meeting a parallel to PD,
through Q, at F.
Now, CV is a given Line, or V is a given point, and therefore,
VF is given in position.
Also,
PQ : KV,
and QF : KV =PD : PC, are
given Ratios;
Or PQ : QF is
a given Ratio.

This Case, therefore, will be determined by the preceding one.

We add, To determine the Triangle, given the Hypothenuse, and the Sum or the Difference, of

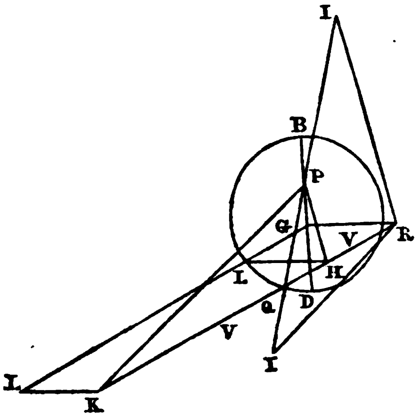
two Lines, to which one Side, and the alternate Segment of the Hypothenuse, by a perpendicular from the right Angle, have given Ratios.

PROBLEM VI.

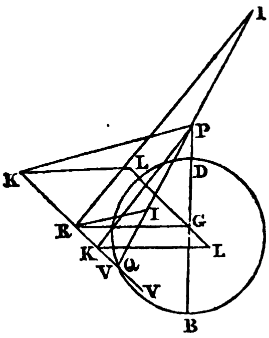
The Line GR is perpendicular to the diameter BD of a given Circle, and P is a given point in BD: to determine a point Q, in the periphery, such, that drawing QR, making a given Angle with GR, and joining P, Q: the Difference or the Sum, of PQ, and a given Line PI, shall have a given Ratio to QR.

ANALYSIS.

Suppose Q to be determined:
Draw IR, and PK parallel thereto, meeting QR at K:
also, draw KL, GL, parallel to RG, RK.
The Ratio IQ: QR is given,
And because of Parallels,
PI : KR = IQ : RQ = PQ : KQ,



Or
PI : KR, and PQ : KQ,
are given Ratios,
and PI is a given Line;
That is,
RK = GL,
is a given Line.
Also,
the point G is given,
and GL is parallel to QR, which makes a given Angle with RG,
Or GL makes a given Angle with GR,
a Line given in position,
The position of GL is, therefore, given;
And,
as GL is given in Length,
The point L is, hence, given.
The Line LK, then, is drawn from a given point, parallel to GR,
a Line, of which the position is given;



Or
LK
is given in position.

WHEREFORE, The Point P is posited in the Diameter BD, of a given Circle, and the Line LK is perpendicular to that Diameter: to draw PQ, to a point Q, in the Periphery, such, that QK being drawn, making a given Angle with LK; the Ratio PQ : QK shall be given.

WHICH IS PROBLEM II. B. II.

COMPOSITION.

Draw GL, making the given Angle with GR, the Ratio PI : GL equal the given one, and let LK be parallel to GR. Then, by Problem II. B. II. determine Q, such, that joining Q, P, and drawing QK, making the given Angle with LK; the Ratio PQ : QK shall be equal PI : GL.

The above Problem would have been more general, and, of course, more interesting to the Geometer, under the following Form.

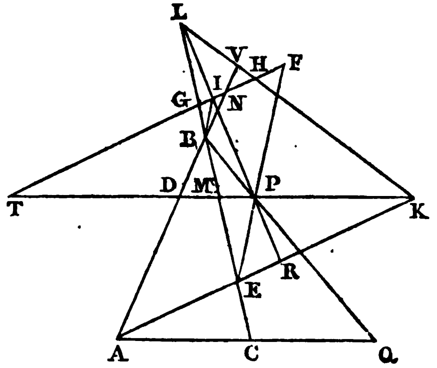
To draw PQ, to Q, in the periphery of the Circle; and QR, making a given Angle with GR, so, that adding to, or taking from, PQ, QR, the given Lines PI, QV; or adding PI to PQ, and taking QV from QR: the Ratio IQ : QV shall be given.

PROBLEM VII.

From a given point P, any how posited, with respect to two Lines BA, BC, given in position, to draw the right Lines PF, PT, to cut off a Segment FT, from another right Line FT, given in position, such, that dividing FT in a given Ratio at I: the Line BI may be of a given Length, and the Segments BD, BE, cut from BA, BC, by PT, PF, may have a given Ratio.

ANALYSIS.

Suppose the position of the Lines PF, PT, to be determined: To FT, apply, from the point B, the Line BI equal the given one,



Then I is a given Point:
Draw PI, which will be given in position ; and the Intersection
L, of CB, PI,
Lines given in position, will be a given point.
Let FT, FP, meet BC at G, E ; and, let a parallel to FT,
through E, meet BA at A, IP at R, and TP at K :
Draw LK, intersecting AB, FT, at V; H.
Because of Parallels,
IF : IT=RE : RK=IG : IH,
is a given Ratio,
and IG is a given Line,
Then,
IH, and, thence, the Point H, are given :
The points L, H, then, are given,
Or the Line LK is given in position.
Draw BP, and let a parallel to KT, through A, meet BC,
BP, at C, Q.
The Ratio BD : BE is given,
And, as AE is parallel to FT, a Line given in position,
The Ratio BA : BE is also given,

That is, BD : BA is a given Ratio;
Now, because of Parallels,
BP : BQ= BD : BA,
Therefore, BP : BQ is a given Ratio;
And consequently, the Point Q, as BP is given in Length, and BQ in position, is a given Point.

The Parallels KP, AQ, then, pass through two given Points P, Q, and meet VK, VA, Lines given in position, at K, A, so, that KA is parallel to FT, another Line given in position: or VK : VA a given Ratio.

THIS IS PROBLEM XI. B. I.

COMPOSITION.

Having drawn the Lines as by the Analysis, let AB, FT, intersect at the point N; the given Ratios being GI : IH and BM : BG.

Take BQ : BP= BN : BM.

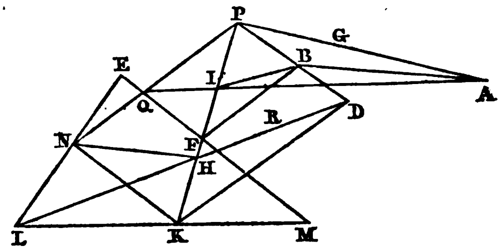
Then draw, by Problem XI. the Parallels PK, QA, so, that VK : VA=VH : VN.

◆

When P is posited *between* the Lines BA, BC, the Problem is the same as Question 594. GENTLEMAN'S DIARY: to which Case, an Analysis and Composition have been given, by MR. WHITE, in the Diary for 1789.

This Problem is also a particular Case of the second Deduction from the XXVI. Problem, of B. I.

When V, L, I, are Points, any how given, in



ANALYSIS.

We will suppose the Point P determined, and the Lines drawn
agreeably to the Problem.
Let BI, a parallel to RD, meet PK, at I, and draw AI.
The Triangles
PAB, PIB,
are, obviously, given in Species,
Or $BP : PA$, and $BP : PI$, are
given Ratios;
Then,
The Ratio $PA : PI$ is given,
and, therefore, AI is given in position.
Let RD, PK, intersect at H, and MF, AI, at the point Q:
Then, Q is a given Point.
Draw PQ, meeting HN, a Parallel to IQ, at N; and join
N, K.
Because of Parallels,
$PB : BD = PF : FK = PI : IH = PQ : QN$,
Or KN is parallel to FQ or MQ:
The Lines KH, KN, HN, therefore,
are parallel to Lines given in position,
Or the Triangle KHN is given in Species,
and consequently,
The Ratio $KH : HN$ is given.

Let MK, RH, intersect at L; and, through N, draw LE.
The Point L, being the Intersection of two Lines given in position, is a given Point;
Also,
The Triangle LKH is given in Species,
and, thence, KH : HL is a given Ratio;
Then,
The Ratio LH : HN is given.
Now, as LHN is a given Angle, LH : HN a given Ratio,
and L a given Point;
It follows,
that the Line LN, or LE, is given in position.

WHEREFORE,

We have given in position, The three Lines AG, ML, EL, and the Point Q: to draw, through Q, the Line PN, meeting AG, EL, two of the Lines given in position, at P, N, so, that drawing PK, NK, parallel to two other Lines given in position, the Point of concourse K, shall be posited in LM, the remaining Line, given in position.

This is A CASE OF PROBLEM I. B. I. See the Solution at the End of that Problem.

From our Deductions, we select an additional Problem.

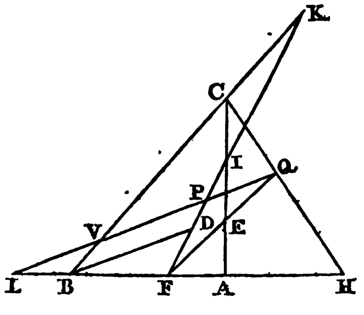
The right Lines HF, FN, DQ, BK, and the Points H, I, in HF, NF, are given in position:

BA, BC, FT: to draw PF, PT, making VD : LE, and FI : IT, equal given Ratios.

The Analysis will differ very little from that given above.

PROBLEM VIII.

To draw, from the given Points F, H, the right Lines FQ, HQ, to Q, a point in the Line LQ, given in position, to meet AI, another right Line given in position, in E, C, so, that the Rectangle of the Segments AE, AC, shall be given.



ANALYSIS.

We will suppose the required Position of FQ, HQ, to be assigned.

Draw FI to AC, LQ, at I, P, making the Angle $FIA=FLQ$; also draw CV to LQ, LA, FP, at V, B, K, making the Angle $CVP=QFP$, and let BD be parallel to LP.

The Angle
$IFE=CVP=BVL$,
And the Angle
$FIE=BLV$;
Then, the Angle
$AEF=ABC$,
Or the Points F, B, C, E, are in the Periphery of a Circle;
And therefore,
the Rectangle $AF.AB=AE.AC$, is given:
Also, AF is a given Line,
That is,
AB, or the Point B,
is given.
Now, the Angle $KBD=KVP=QFP$, and the Angle
$FQP=VKD$,
Or the Triangles BDK, QPF, are equiangular,
Having
$DK:QP=DB:PF$,
Or
$DK:QP$
is a given Ratio.

Whence, The three Lines AC, DK, PQ, and the two points B, H, are given in position; to draw BC, HC, to C, a point in AC, such, that meeting DK, PQ, at K, Q; the Segments DK, PQ, shall have a given Ratio.

A Particular Case of Problem XXVI. B. I.

COMPOSITION.

Take the point B, and draw FP, BD, as in the Analysis. Then, by Problem XXVI. B. I. drawing HC, BC, to intersect at C, in AI, so, that meeting LP, FP, at Q, K, the Ratio PQ : DK may be equal PF : BD, it is done.

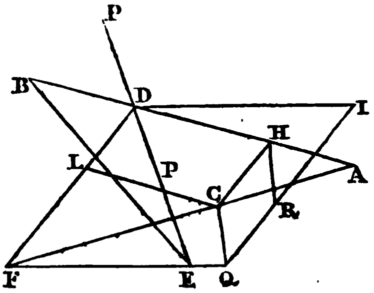
The following deduced Problem will not be uninteresting.

Through two given points, to describe a Circle, intersecting a right Line, given in position, in two points, such, that Chords, drawn through the two points of Intersection, and two other given points, shall intersect another right Line, given in position, in the same Point.

PROBLEM IX.

The right Lines BD, BE, QR, and the point P, are given in position; also, the points Q, R, in the Line QR, and the point H, in the Line

BD, are given: to draw the Lines PE, QE, to intersect each other in the Line BE, so, that from D, the point of Intersection of PE, BD, drawing DI parallel to EQ, to QR at I: the Ratio of the Segments HD, RI, shall be given.



ANALYSIS.

Let PE, QE, be supposed drawn, in the required position:
Draw RH, and QC, HC, parallel to RH, RQ.
The points H, R, Q, are given,
And, therefore,
QC, HC,
drawn parallel to RH, RQ,
are given in position,
Or C is a given point.
Draw DF, CL, parallel to QI, DB, meeting QE, DF, at F, L;
and, let FC intersect BD or BH, at A.
Because of parallel Lines,

That is,
The Points D, P, in the Line AC, are given points.
Also,
The Angle ACB, and thence BCD, are given,
and BC : CD is a
given Ratio,
Or the Angle CDB is a given Angle,
Therefore, its Equal, the Angle CPQ, is a given Angle;
Or PQ is given in position.
Draw DR, QR, parallel to CQ, CD, and join P, R.
The Ratios
BD : DC= BD : RQ, and BD : QP= AD : AP,
Are given,
Consequently, QR : QP is a given Ratio;
and since, by Parallels,
The Angle PQR =BDC= a given Angle,
It follows, that QPR is a given Angle:
That is,
PR
is given in position,
and
PQ : PR
is a given Ratio.

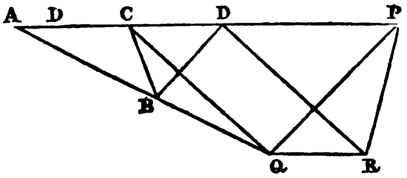
CONSEQUENTLY,

The Lines PQ, PR, and the Points A, D, are, given in position, and our Problem is, to draw AQ, DR, to include a given Angle, so, that meeting the Lines given in position at Q, R, the Ratio PQ : PR shall be given.

Which is A CASE OF PROBLEM XVII. B. I.

PROBLEM X.

Given, the vertical Angle, the Sum or the Difference, of one Side and a Line to which the other Side has a given Ratio, and the Ratio of the Segments of the Base, by a right Line from the vertex, making a given Angle with the Base, to determine the Triangle.



ANALYSIS.

The Triangle ACB being supposed the required one: From C, the given vertical Angle, draw CQ, making the given Angle CQA, with the Base AB: let CD be the Line to which the Side CB has the given Ratio, draw DB, and QP parallel thereto.

The Line AD is given,
and
AP : PD= AQ : QB,
is a given Ratio,
Or AP is a given Line;

That is,
The Points D, P, in the Line AC, are given points.
Also,
The Angle ACB, and thence BCD, are given,
and BC : CD is a
given Ratio,
Or the Angle CDB is a given Angle,
Therefore, its Equal, the Angle CPQ, is a given Angle;
Or PQ is given in position.
Draw DR, QR, parallel to CQ, CD, and join P, R.
The Ratios
BD : DC= BD : RQ, and BD : QP= AD : AP,
Are given,
Consequently, QR : QP is a given Ratio;
and since, by Parallels,
The Angle PQR =BDC= a given Angle,
It follows, that QPR is a given Angle:
That is,
PR
is given in position,
and
PQ : PR
is a given Ratio.

CONSEQUENTLY,

The Lines PQ, PR, and the Points A, D, are, given in position, and our Problem is, to draw AQ, DR, to include a given Angle, so, that meeting the Lines given in position at Q, R, the Ratio PQ : PR shall be given.

Which is A CASE OF PROBLEM XVII. B. I.

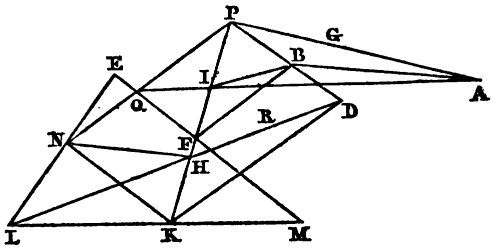
The preceding Problem is a generalised Form, which we have given to the VII. and IX. in the Appendix to SIMPSON's Algebra.

The following Enunciation will furnish an additional Exercise.

Given the Ratio of the Segments of the Base, by a Line from the Vertex, making a given Angle with the Base, the Sum or the Difference, of two Lines, to which the Sides have given Ratios, and the Vertical Angle : to determine the Triangle.

PROBLEM XI.

The Lines AG, AB, RD, MF, MK, are given in position : to determine the Point P, in one of the Lines, as AG, such, that drawing PBD, PFK, parallel to two other Lines, given in position, meeting AB, RD, MF, MK, at B, D, F, K : the Lines BF, DK, passing through the Points B, F ; and D, K ; shall be parallel to each other.



ANALYSIS.

We will suppose the Point P determined, and the Lines drawn
agreeably to the Problem.
Let BI, a parallel to RD, meet PK, at I, and draw AI.
The Triangles
PAB, PIB,
are, obviously, given in Species,
Or BP : PA, and BP : PI, are
given Ratios;
Then,
The Ratio PA : PI is given,
and, therefore, AI is given in position.
Let RD, PK, intersect at H, and MF, AI, at the point Q:
Then, Q is a given Point.
Draw PQ, meeting HN, a Parallel to IQ, at N; and join
N, K.
Because of Parallels,
PB : BD=PF : FK=PI : IH= PQ : QN,
Or KN is parallel to FQ or MQ:
The Lines KH, KN, HN, therefore,
are parallel to Lines given in position,
Or the Triangle KHN is given in Species,
and consequently,
The Ratio KH : HN is given.

Let MK, RH, intersect at L, and, through N, draw LE.
The Point L, being the Intersection of two Lines given in position, is a given Point;
Also,
The Triangle LKH is given in Species,
and, thence, KH : HL is a given Ratio;
Then,
The Ratio LH : HN is given.
Now, as LHN is a given Angle, LH : HN a given Ratio,
and L a given Point;
It follows,
that the Line LN, or LE, is given in position.

WHEREFORE,

We have given in position, The three Lines AG, ML, EL, and the Point Q: to draw, through Q, the Line PN, meeting AG, EL, two of the Lines given in position, at P, N, so, that drawing PK, NK, parallel to two other Lines given in position, the Point of concourse K, shall be posited in LM, the remaining Line, given in position.

This is A CASE OF PROBLEM I. B. I. See the Solution at the End of that Problem.

◆

From our Deductions, we select an additional Problem.

The right Lines HF, FN, DQ, BK, and the Points H, I, in HF, NF, are given in position:

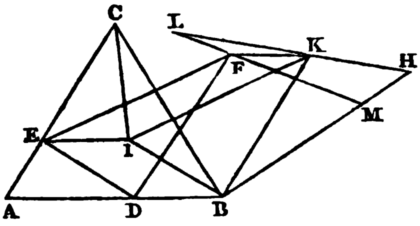
to draw NQK, parallel to another Line given in position, meeting FN, DQ, BK, at N, Q, K, so, that making HM : N1, and FV : FM, equal to given Ratios, and joining K, V : A Parallel to KV, through Q, shall pass through a given point P.

PROBLEM XII.

Let AB, AC, be two Lines, given in position; B, C, given points in those Lines, and LM another Line, given in Position: to describe a Triangle DFE, given in Species, having its angular points D, E, upon AB, AC, and the remaining Angle F upon the Line LM, so, that the Ratio of the Segments BD, CE may be given.

ANALYSIS.

Suppose the position of the Triangle DEF to be assigned, as required by the Problem.



Draw BI, EI, parallel to DE, DB, and join the points C, I.

By parallels,

BD= IE;

Also,

The Ratio BD : EC=IE : EC, is given,

and IEC a given Angle,

Or the Angle ECI is given:

The point C, also, is given,

then, CI is given in position.

Draw BK, IK, parallel to DF, EF; also, draw BC, BH, KH,

making the Angle CBH= IBK, and BKH=BIC:

Let HK meet ML at L, and join K, F.

Evidently,

BH

is given in position;

B being a given Point, BC a Line given in position,

and CBH=IBK =EDF equal

a given Angle.

Also,

CBI, and HBK,

are equiangular Triangles,

Having IB : BK =CB : BH=ED : DF a given Ratio,

and CB is a given Line,

Then, BH is given in Length, as well as in position;

meeting DK, PQ, at K, Q; the Segments DK, PQ, shall have a given Ratio.

A PARTICULAR CASE OF PROBLEM XXVI. B. I.

COMPOSITION.

Take the point B, and draw FP, BD, as in the Analysis. Then, by Problem XXVI. B. I. drawing HC, BC, to intersect at C, in AI, so, that meeting LP, FP, at Q, K, the Ratio PQ : DK may be equal PF : BD, it is done.

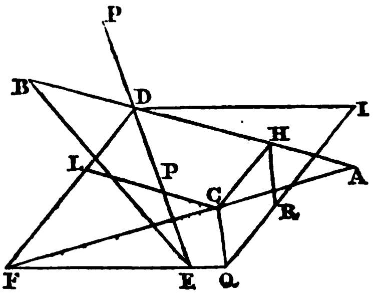
The following deduced Problem will not be uninteresting.

Through two given points, to describe a Circle, intersecting a right Line, given in position, in two points, such, that Chords, drawn through the two points of Intersection, and two other given points, shall intersect another right Line, given in position, in the same Point.

PROBLEM IX.

The right Lines BD, BE, QR, and the point P, are given in position; also, the points Q, R, in the Line QR, and the point H, in the Line

BD, are given: to draw the Lines PE, QE, to intersect each other in the Line BE, so, that from D, the point of Intersection of PE, BD, drawing DI parallel to EQ, to QR at I: the Ratio of the Segments HD, RI, shall be given.



ANALYSIS.

Let PE, QE, be supposed drawn, in the required position:
Draw RH, and QC, HC, parallel to RH, RQ.
The points H, R, Q, are given,
And, therefore,
QC, HC,
drawn parallel to RH, RQ,
are given in position,
Or C is a given point.
Draw DF, CL, parallel to QI, DB, meeting QE, DF, at F, L;
and, let FC intersect BD or BH, at A.
Because of parallel Lines,

$CL = HD$, $DF = IQ$, and $DL = HC = RQ$;
Consequently,
$LF = RI$:
Also,
$HD : RI = CL : LF = AH : HC$,
is a given Ratio, and HC is a given Line,
Hence, HA, or the point A, is given;
That is,
AC is given in length, AF is given in position,
and
$AF : AD = AC : AH$,
is a given Ratio.

WHENCE,

The points Q, P, and the lines BE, AF, AD, are given in position: to inflect QE, PE, to a Point E, in BE, so, that intersecting AF, AD, at F, D; the Ratio of the Segments AF, AD, shall be given.

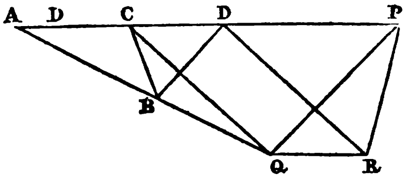
Which Problem is only a PARTICULAR CASE OF THE XXVI. B. I.

The COMPOSITION may now be omitted, for the pursuit and consequent advantage of the Student.

Retaining the Data in the Problem, To draw PE, QE, so, that the Sum or the Difference, of HD, RI: or the Sum, or the Difference, of two Lines, to which HD, RI, have given Ratios, shall be equal a given Line.

PROBLEM X.

Given, the vertical Angle, the Sum or the Difference, of one Side and a Line to which the other Side has a given Ratio, and the Ratio of the Segments of the Base, by a right Line from the vertex, making a given Angle with the Base, to determine the Triangle.



ANALYSIS.

The Triangle ACB being supposed the required one: From C, the given vertical Angle, draw CQ, making the given Angle CQA, with the Base AB: let CD be the Line to which the Side CB has the given Ratio, draw DB, and QP parallel thereto.

The Line AD is given,

and

AP : PD = AQ : QB,

is a given Ratio,

Or AP is a given Line;

That is,
The Points D, P, in the Line AC, are given points.
Also,
The Angle ACB, and thence BCD, are given,
and BC : CD is a
given Ratio,
Or the Angle CDB is a given Angle,
Therefore, its Equal, the Angle CPQ, is a given Angle;
Or PQ is given in position.
Draw DR, QR, parallel to CQ, CD, and join P, R.
The Ratios
BD : DC= BD : RQ, and BD : QP= AD : AP,
Are given,
Consequently, QR : QP is a given Ratio;
and since, by Parallels,
The Angle PQR =BDC= a given Angle,
It follows, that QPR is a given Angle:
That is,
PR
is given in position,
and
PQ : PR
is a given Ratio.

CONSEQUENTLY,

The Lines PQ, PR, and the Points A, D, are, given in position, and our Problem is, to draw AQ, DR, to include a given Angle, so, that meeting the Lines given in position at Q, R, the Ratio PQ : PR shall be given.

Which is A CASE OF PROBLEM XVII. B. I.

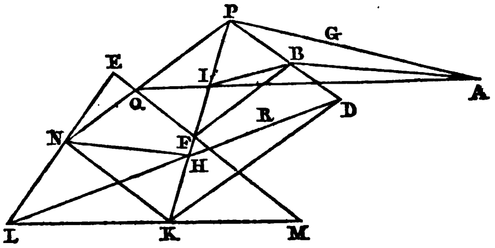
The preceding Problem is a generalised Form, which we have given to the VII. and IX. in the Appendix to SIMPSON's Algebra.

The following Enunciation will furnish an additional Exercise.

Given the Ratio of the Segments of the Base, by a Line from the Vertex, making a given Angle with the Base, the Sum or the Difference, of two Lines, to which the Sides have given Ratios, and the Vertical Angle: to determine the Triangle.

PROBLEM XI.

The Lines AG, AB, RD, MF, MK, are given in position: to determine the Point P, in one of the Lines, as AG, such, that drawing PBD, PFK, parallel to two other Lines, given in position, meeting AB, RD, MF, MK, at B, D, F, K: the Lines BF, DK, passing through the Points B, F; and D, K; shall be parallel to each other.



ANALYSIS.

We will suppose the Point P determined, and the Lines drawn agreeably to the Problem.

Let BI, a parallel to RD, meet PK, at I, and draw AI.
The Triangles
PAB, PIB,
are, obviously, given in Species,
Or BP : PA, and BP : PI, are
given Ratios;
Then,
The Ratio PA : PI is given,
and, therefore, AI is given in position.
Let RD, PK, intersect at H, and MF, AI, at the point Q:
Then, Q is a given Point.
Draw PQ, meeting HN, a Parallel to IQ, at N; and join
N, K.
Because of Parallels,
PB : BD = PF : FK = PI : IH = PQ : QN,
Or KN is parallel to FQ or MQ:
The Lines KH, KN, HN, therefore,
are parallel to Lines given in position,
Or the Triangle KHN is given in Species,
and consequently,
The Ratio KH : HN is given.

Let MK, RH, intersect at L; and, through N, draw LE.
The Point L, being the Intersection of two Lines given in position, is a given Point;
Also,
The Triangle LKH is given in Species,
and, thence, KH : HL is a given Ratio;
Then,
The Ratio LH : HN is given.
Now, as LHN is a given Angle, LH : HN a given Ratio,
and L a given Point;
It follows,
that the Line LN, or LE, is given in position.

WHEREFORE,

We have given in position, The three Lines AG, ML, EL, and the Point Q: to draw, through Q, the Line PN, meeting AG, EL, two of the Lines given in position, at P, N, so, that drawing PK, NK, parallel to two other Lines given in position, the Point of concourse K, shall be posited in LM, the remaining Line, given in position.

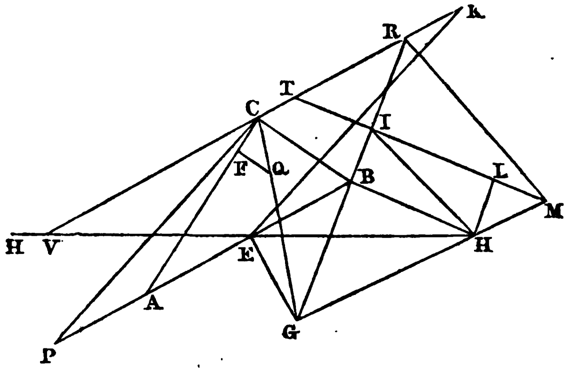
This is A CASE OF PROBLEM I. B. I. See the Solution at the End of that Problem.

* * *

From our Deductions, we select an additional Problem.

The right Lines HF, FN, DQ, BK, and the Points H, I, in HF, NF, are given in position;

Lines, to which the Base, and a Line from the vertex, making a given Angle with the Base, have given Ratios; to determine the Triangle.



ANALYSIS.

Let ACB be supposed the required Triangle; Q, the center, and QF, the radius, of the inscribed Circle: join Q, and C, the Vertex.
Then CQ is given.
Produce CQ, to meet a Perpendicular to the Base, from E, the point of its bisection, at G; draw CV parallel to BA:
Let EH, and EV, in EH produced, be the Lines to which the Base, and the Line PC, from the vertex, have the given Ratios:
Draw HB, HG; also EK, parallel to PC.
Now, HV is given;
Also,
as EV has a given Ratio to PC or to EK,
and the Angle EKV equal a given one,
The Angle HVC=HEB is given;

Or VC is given in position.
Again,
The Ratios HE : EB, GE : EB ; and, consequently,
the Ratio HE : EG, are given,
and the Angles GEH, BEH, given Angles ;
Or
The Angles EHG, EHB, are given :
and, therefore, HG, HB, are given in position.
Let GB meet VC at R, take BI =QC, draw IT, HL, parallel to HB, GB : the former meeting VR, HL,
GH, at T, L, M ; and draw HI, MR.
The Angle MHL=HGB, is given,
and H is a given Point,
Or the Line HL is given in position :
The Line HL is given, also, in Length,
For,
because of parallels,
HL=BI=QC,
a given Line ;
The point L, then, is given :
That is, TI passes through a given Point, and is parallel to a Line given in position,
Therefore, TI is given in position ;
and, consequently, M, being the Point of concourse of two Lines,
given in position, is a given Point.
Moreover,
By a known Property of Triangles,
GB= GQ,
and, as BI, QC, are equal,
It follows, that
GI =GC.
Now, because of the equal Angles
GCB, and GRC,

GC : GB = GR : GC = GI : GB = GR : GI = GM : GH,
because of the Parallels
IM, BH:
That is, MR, HI, are parallel.

CONSEQUENTLY,

The Points M, H, and the Lines TR, TI, are given in position: and, we have to draw the Parallels MR, HI, so, that RI shall be parallel to HL, another Line given in position:
Or TR : TI a
given Ratio.

This is A PARTICULAR CASE OF OUR XI. PROBLEM, B. I.

The following is added, as having a more general Form than the preceding.

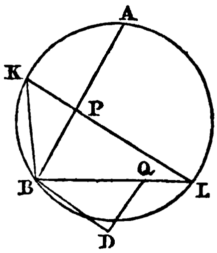
To determine the Triangle, there are given, the Sum or the Difference, of two Lines, to which the Base, and a Line drawn from the Vertex, making a given Angle with the Base, have given Ratios; also, two Lines, to which the Diameter of the inscribed Circle, and the Difference between the Sum of the Sides and the Base, have given Ratios.

Problem XV. is a general Enunciation which we have given to the 64. Question, BURROW'S DIARY, proposed and solved, by the ingenious Mr. CRAKELT.

Of Reuben Burrow, that distinguished, elegant Geometer, and able Mathematician, I am not aware that, even the most humble Memoir has yet been given.

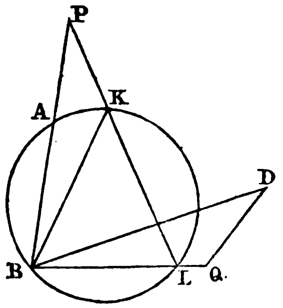
PROBLEM XVI. ✓

To describe, through two given Points A, B, a Circle, intersecting a right Line KL, given in position, in K, L, so, that drawing BK, BL; the Angle KBL shall be equal a given Angle.



ANALYSIS.

Suppose the required Circle to be described.
Draw AB, intersecting KL in P: also, draw BD, DQ, making the Angle ABD = KBL, BD = BP, and the Angle BDQ = BPK.
The Line AB is given in position, and ABD is a given Angle,
Or BD is given in position:
Also, BD being equal to BP, a given Line,
the point D is, thence, given.



Again,
BDQ=BPK,
is a given Angle, DB is given in position, and D, a given point,
Or DQ is given in position.
Let DQ, BL, meet at Q.
The Angle
KBL=PBD,
consequently, Angle
KBP=QBD;
And, therefore, the Triangles
KPB, QDB,
having, also, the Angle KPB= QDB, are equiangular,
Or
KP : PB= QD : DB,
and
DB=BP;
Then,
PK=QD.
Now, as PK= QD, it follows,
that PL . PK=PL . QD= AP . PB, is a
given Rectangle.

WHEREFORE,

The Lines PL, DQ, and the point B, are given in position: and, we have to draw BLQ, meeting PL, DQ, at L, Q, so, that the Rectangle PL. DQ, shall be equal a given Space.

WHICH IS PROBLEM I. B. II.

◆

This Problem is the same as the 228. Question, LEYBOURN'S REPOSITORY, original Series.

The following Problems may, hence, be solved.

Having a Point and a right Line, given in position: to draw, from the given point, two Lines, including a given Angle, that shall cut off, from the Line given in position, two Segments, between the points of intersection, and a given point, or given points, in that Line, the Rectangle of which, shall be given.

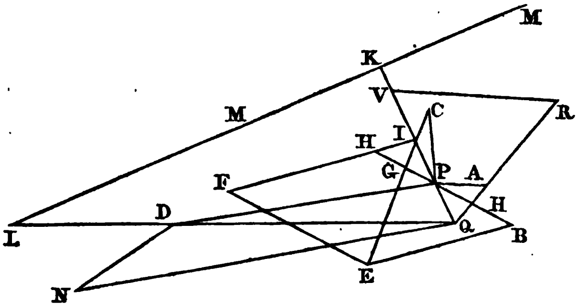
Two given Circles, a Diameter AB, of one of them, and a Chord KL, of the other, are given in position: to assign, in AB, the Points P, Q, such, that

$$AP : PB = AQ : QB,$$

and, that PK, QL, shall intersect in the Periphety of the Circle KL.

PROBLEM XVII.

Having four right Lines, given in position: to determine a Point, from which, if four other right Lines are drawn, meeting the former in given Angles, the Sum or the Difference, of two of the Lines so drawn, and the Sum or the Difference, of the Rectangles, under the remaining two, and two given Lines, shall be given.



ANALYSIS.

Let the Lines, from the required Point P, meet QA, EB, EC, ND, the Lines given in position, at the Points A, B, C, D.

In BP,

Take PH = PC, draw EF, HF, parallel to BH, BE:

Let PH, FH, meet EC at G, I; and draw IP.

The point E is given,
and EF is parallel to BH, which makes a given Angle
with EB,
Or EF is given in position ;
Also, because of Parallels,
$EF = BH = BP \pm PH = BP \pm PC$
equal a given Line ;
That is,
the point F is given.
The Line FI, then, passes through a given point,
and is parallel to EB, a Line given in position ;
consequently,
I is a given point.
Now, the Triangle CGP is given in Species,
Or
$CP : PG = HP : PG$,
is a given Ratio ;
also, IH, IG, are
given in position, and HP is parallel to FE, a Line given
in position,
Or the position of IP is given.
Let IP meet AQ at Q, take QV, QK, equal the two given
Lines ; draw VR ; KL, QN ; parallel to PA ; PD ; and
make $KM = VR$.
The points V, K, Q, are given, and VR, KL, QN, are parallel to Lines given in position,
therefore, VR, KL, QN, are given in position :
Also, as the position of QR, DN, is given,
$VR = KM$,
is a given Line,
And M, N, are given points.
Moreover, by parallel Lines,
$QP : PA = QV : VR = QV : KM$,

And

$QP : PD = QK : KL$,

Then,

$QP . KM = PA . QV$, and $QP . KL = PD . QK$:

Whence, $QP . ML = PA . QV \pm PD . QK$, is a given Rectangle,

and $QP : ND = QP . ML : ND . ML$ is a given Ratio:

That is,

$ND . ML$

is a given Rectangle.

HENCE,

The Lines ND, ML, and the point Q, are given in position: and, it is required to draw QDL, making the Rectangle of the Segments ND, ML, equal a given Space.

THIS IS PROBLEM I. B. II.

A particular Case, of the above general Problem, forms the XXIII. Problem of NEWTON's Univer. Arith. where an Analytical Solution is given: see Wilder's Edit. of 1769.

The same Case was proposed in the LADIES' DIARY, Ques. 831. and three Geometrical Solutions inserted in the Diary for 1784.

A Problem, still more general, and more interesting than the preceding, may be determined by the same method of Analysing.

We present the following, as a subject of continued Inquiry.

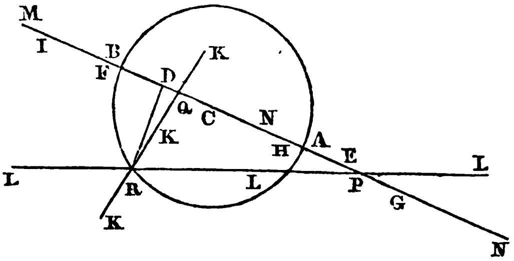
Given in position, four right Lines: to assign the position of a Point such, that drawing, from that point, four other right Lines, meeting those given in position in given Angles; the Sum, or the Difference, of two Lines, to which two of the Lines, so drawn, have given Ratios, shall be equal a given Line; and, the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the two remaining Lines drawn from the Point assigned, have given Ratios, equal a given space.

PROBLEM XVIII.

To draw, from the points P, Q, posited in the Diameter AB, of a given Circle, the right Lines PR, QR, to a point R, in the Periphery, such, that QR shall have a given Ratio to the Difference, or to the Sum, of PR and a given Line PL.

ANALYSIS.

Suppose the position of the Point R to be assigned. Draw RD at right Angles to AB, and let C be the center of the Circle.



The Ratio $RL : RQ$ is given,
That is, $RL^2 : RQ^2$, or $(PL \pm PR)^2 : RQ^2$, is a given Ratio.
Make $AH : HB$ equal $AP : PB$, and
$PE = EH$;
Also, $BI : IA$ equal $BQ : QA$, and
$QF = FI$:
Then, the Points H, E, I, F, are evidently given.
Take $2PC : PL = PL : EG$,
Then, since $2PC . EG =$ a given Square; PC, or $2PC =$ a given Line, and E a given point;
EG, and the point G, are, thence, given.
Also, by a Theorem, universally known,
$PR^2 = 2PC . DE$, and $RQ^2 = 2QC . DF$;
consequently,
$(PL \pm PR)^2 = \pm 2PC . DG \pm 2PL . PR$:
Take $PR : GN = PL : PC$, a given
Ratio,
Then
$PL . PR = PC . GN$,
and therefore,
$(PL \pm PR)^2 = 2PC . DN$;
That is, $2PC . DN : 2QC . DF = PC . DN : QC . DF$, is a given Ratio.

Let the Ratio

$PC . DN : QC . DF = PC : CM = PC . DN : CM . DN$,

Then,

CM is a given Line, and $QC . DF = CM . DN$,

Or

$DF : DN = CM : CQ$,

is a given Ratio.

Moreover, since $GN : PR = PL : PC$, is a given Ratio,

It follows, that

$GN^2 : PR^2 = PL^2 : PC^2 = GN^2 : 2PC . DE$;

Or

$GN^2 : PC . DE$

also, is a given Ratio.

WHENCE,

In the Line MN, we have given, the Points P, C, E, F, G: to determine, in MN, the two Points D, N, such, that $DF : DN$, and $GN^2 : PC . DE$, shall be given Ratios.

Which is PROBLEM III. B. II.

This Problem was proposed and solved, in the MATHEMATICAL COMPANION: See page 48. No. VIII.

To the preceding, without requiring an Analysis materially different, we may give the following more general Enunciation.

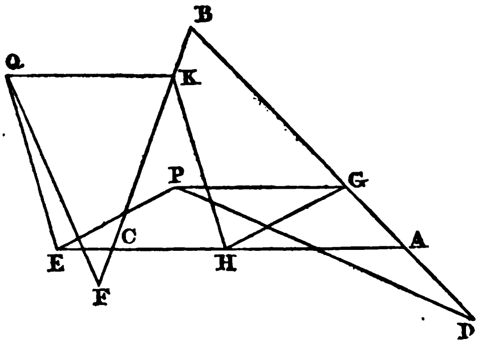
To draw PR, QR, to R, in the Periphery of the given Circle, such, that adding to, or taking from, PR, QR, the given Lines PL, QK; or, adding PL to one, and taking QK from the other: the Sums, or the Differences; or the Sum and Difference, shall have, to each other, a given Ratio.

PROBLEM XIX.

The right Lines AB, AC, BC, are given in position, and D, E, F, are given Points, in those Lines: to determine the Points G, H, K, in the same Lines, such, that drawing GH, KH, the Angle GHK shall be equal a given Angle, and the Segments DG, EH, FK, shall have, to each other, given Ratios.

ANALYSIS.

The position of HG, HK, we suppose to be determined. Draw EP, EQ, parallel to HG, HK; also, GP, KQ, parallel to HE: and join the Points D, P; and F, Q.



The Ratios DG : EH, and EH : FK, are given;
and, because of Parallels,
EH = PG = QK,
Or
PG : GD, and QK : KF,
are given Ratios:
That is,
as DGP, FKQ, are given Angles; and D, F, given Points;
the Lines
DP, FQ,
are given in position.
Again,
the Triangles GPD, KQF, are given in Species,
Or
GP : PD = KQ : PD, and KQ : QF,
are given Ratios:
Consequently, DP : FQ is a given Ratio.
Moreover, E is a given
Point,

And, by parallel Lines,
PEQ = HEK,
is a given Angle.

WHEREFORE,

The Lines DP, FQ, and the Point E, are given in position: to draw EP, EQ, including a given Angle, and meeting DP, FQ, at P, Q, so, that the Ratio of DP to FQ may be given.

THIS IS THE XVI. PROBLEM, B. I.

* * *

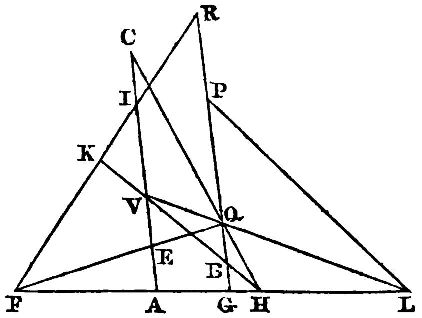
The above Problem, we have taken from the MATHEMATICAL COMPANION: see page 52. No. IV. Also, PROFESSOR WALLACE'S Paper on PORISMS, in the IV. Vol. Edinburgh Transactions.

We give a deduced Problem, as an Exercise.

From two given Points, to draw two Lines, including a given Angle, and meeting two Lines, given in position, so, that drawing, from the Points of Intersection, two Lines, parallel to two other Lines, given in position; the two Lines, last drawn, shall have their Point of concourse, in another right Line, given in position.

PROBLEM XX.

Given in position, the four right Lines FR, LP, KH, HF: to draw the right Line GR, parallel to a Line given in position, to meet the Lines given in position, at R, P, B, G, so, that the Rectangle of the Segments GR, GB, shall have a given Ratio to the Square of the Segment GP.



ANALYSIS.

The Line GR being supposed drawn, take, therein, the Point Q, such, that

$$GR : GQ = GQ : GB,$$

Or

$$GR . GB = GQ^2.$$

The Ratio $GR \cdot GB : GP^2 = GQ^2 : GP^2$, is given,
and, therefore,
$GQ : GP$, or $GQ : QP$,
is a given Ratio.
The Lines LG, LP, are given in position,
GP is divided in a given Ratio at Q, and is drawn parallel to
a Line given in position:
The Point Q, therefore,
is posited in a right Line, the position, of which, may be assigned.
Draw LQ:
Then, LQ is the Locus of Q, and is, therefore, given in position.
Let LQ meet HK at V, draw HQ, FQ; and, through V, draw
AC, parallel to GR, meeting FL, FQ, FR, HQ, at A, E, I,
C.
The Lines LQ, HK, are given in position,
Or V, their point of concourse, is a given Point;
Then, AC,
passing through V, and parallel to a Line
given in position,
is also given in position;
hence, obviously,
the Points A, V, I, in AC, are given:
That is,
AV, AI, and, consequently, the Rectangle $AV \cdot AI$, are given.
Now, because of parallel Lines,
$GB : GQ = GQ : GR = AV : AC = AE : AI$,
Or
$AC \cdot AE = AV \cdot AI$,
a given Rectangle.

The Points F, H, and the Lines VL, VI, then, are given in position: to draw FQ, HQ, intersecting at Q, in LV, and meeting VI, at E, C, so, that the Rectangle of the Segments AC, AE, shall be given.

WHICH IS PROBLEM VIII. B. II.

We here present, for Solution, the following Problems.

The Lines FR, LP, KH, HF, are given in position: to draw GR, parallel to another Line given in position, meeting FR, LP, KH, HF, at R, P, B, G, so, that the Sum, or the Difference, of two Lines, to which GR, GB, have given Ratios, shall have a given Ratio to the Segment GP.

The Lines are posited as in the preceding: to draw GR, parallel to a Line given in position, so, that adding a given Line to each of the Segments GR, GB, GP; or taking a given Line from each of them: the Rectangle under two of the Sums, or under two of the Differences, shall have a given Ratio to the Square of the remaining Sum or Difference.

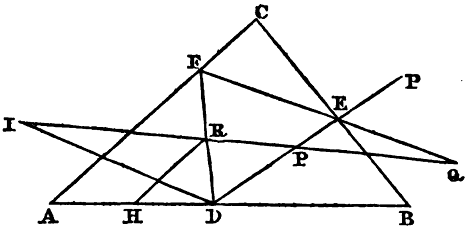
Or so, that

adding a given Line to GR, GB, and taking a given Line from GP; or taking a given Line from GR, GB, and adding one to GP; the

Rectangle of the Sums, or of the Differences, shall have a given Ratio to the Square of the Difference, or of the Sum, of GP and the given Line.

PROBLEM XXI.

To inscribe, in a given Triangle, another Triangle, such, that each Side, of the inscribed Figure, shall pass through a given Point.



ANALYSIS.

Let the Sides DE, EF, FD, of the Triangle DEF, pass through the given Points P, Q, R : the angular Points D, E, F, being upon the Sides AB, BC, CA, of the given Triangle ABC.

Draw QR; also, RH, DI, parallel to CA, QF, meeting AB, QR, at H, I.

The Points

Q, R, A,

and the Lines AB, AC; and consequently RH, a parallel to AC,

are given in position:

Or H is a given point, and QR, AH, given Lines.

Also,

because of parallel Lines,

RF : RD = RQ : RI = AH : HD,

and alternating,

RI : DH = RQ : AH,

a given Ratio.

WE HAVE, THEREFORE, GIVEN IN POSITION,

The Point P, and the Lines BC, BH, QR: to draw DE, through P, meeting BH, BC, at D, E, so, that drawing QE, and DI parallel to QE, meeting QR at I: the Ratio RI : DH may be given.

THIS IS PROBLEM IX. B. II.

MR. LOWRY has given an elegant and general Solution to the above, in the MATHEMATICAL REPOSITORY; See Page 117. Vol. I. New Series.

See, also, Question 155. MATHEMATICAL COMPANION, No. XIII.

This Problem furnishes several Deductions, from which, the following is selected.

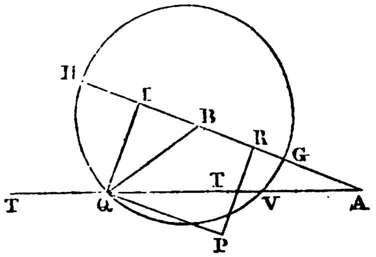
The Lines BN, DL, BM, and the points P, R, Q, are given in position: to draw the Parallels PN, RL, meeting BN, DL, at N, L, so, that NL, and QM, parallel to NL, being drawn: the Lines PN, QM, shall meet BM, at the same Point M.

PROBLEM XXII.

To draw, from two Points, posited in the Diameter of a given Circle, two right Lines, to a point in the Periphery, such, that the Difference of the Lines, so drawn, shall be equal to a given Line.

ANALYSIS.

Suppose AQ, BQ, drawn from the given Points A, B, in the Diameter GH, of the given Circle GQH, to the Point Q, in the Periphery, making the Difference of AQ, BQ, equal to the given Line AT.



Draw QI perpendicular to GH.

And, let R and V, be the Points of bisection, of AB and AT.

Now, $AQ - QB = AT$,

also,

$AQ + QB = 2VQ$,

and,

$AQ^2 - QB^2 = AI^2 - IB^2 = VQ \,.\, 2AT = IR \,.\, 2AB$:

That is,

$VQ : IR = AB : AT$.

Draw RP, QP, parallel to IQ, IR.

Then,

R is a given Point, RP is perpendicular to GH, and

$QP = IR$:

Or,

$VQ : QP = AB : AT$,

a given Ratio.

WHEREFORE,

In the Diameter GH, of the given Circle GQH, the given Point A is posited; and RP is a right Line, perpendicular to GH: to draw AQ, to a point Q, in the Periphery, such, that drawing QP, at right Angles to RP, and taking AV= a given Line; the Ratio VQ : QP shall be given.

WHICH IS PROBLEM VI. B. II.

* * *

This Analysis will be applicable, when the *Sum* of AQ, BQ, is given.

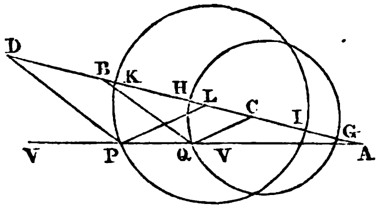
The above is the same as Problem L. Part II. SIMPSON'S Exercises, DR. HUTTON'S Edition.

The preceding is also a particular Case of Problem XVIII. B. II.

For, as the Sum, or the Difference, of AQ, BQ, is given: having taken AT equal to that Sum or Difference, it will only remain, to draw AQ, BQ, so, that QT =QB, or QT to QB in the Ratio of equality.

When we give the Problem a more general Form, and require AQ, BQ, to be drawn, so, that the Sum, or the Difference, of two Lines, to which AQ, BQ, have given Ratios, shall be equal a given Line.

The Analysis, by the Application of the XVIII. as an Auxiliary Problem, will be equally simple with the foregoing.



For, let AP, PV, be the Lines to which AQ, QB, have the given Ratios, and C the centre of the Circle.

Draw QC; also, PL, PD, parallel to QC and QB.

Then, AV is a given Line.

And, because of Parallels,

AQ : AP = AC : AL = AB : AD = CQ : LP,

are given Ratios, and AC, AB, CQ, given Lines:

Therefore, AL, AD, LP, and the Points L, D, are given.

Let the Circle, to Centre L and radius LP, be described.

Then,

LP

is a given Circle.

Also, QB : PD, and QB : PV, are given Ratios,

That is, PD : PV is a given Ratio.

HENCE,

It only remains, To draw, from the given Points

A, D, situated in the Diameter IK, of the given Circle IPK, the Lines AP, DP, to P, in the Periphery, so, that taking, in AP, the Part AV, of a given Length, the Ratio PD : PV shall be given.

WHICH IS THE XVIII. OF B. II.

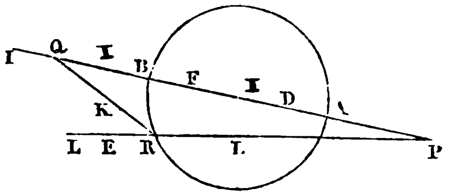
The general Form of Problem XXII. may be otherwise enunciated: as,

To determine the Triangle, there are given, the Base, a Line, from the Vertex, dividing the Base in a given Ratio, and the Sum, or the Difference, of two Lines, to which the Sides have given Ratios.

PROBLEM XXIII.

From two given Points, any how posited, in the Diameter of a given Circle: to draw two right

Lines, intersecting at some Point in the Periphery, such, that the Sum, or the Difference, of the Rectangles, under these two Lines, and two given Lines, may be equal to a given Magnitude.



ANALYSIS.

Let P and Q, be the two given points, in the Diameter AB, of the given Circle ARB, and R, the Point in the Periphery, to which PR, QR, are required to be drawn, making the Sum, or the Difference, of the Rectangles under PR, QR, and the given Lines PD, PF, equal a given Magnitude.

Take

$$PI.PD$$

equal the given Magnitude:

Then,

$$PR.PD \pm QR.PF = PI.PD.$$

In PR, take the Point L, so, that

$$QR : QL = PD : PF,$$

then,

$$QR.PF = RL.PD;$$

and, consequently,

$$PR.PD \pm RL.PD = PI.PD.$$

That is,

$PR \pm RL = PI = PL$

equal a given Line ;

and,

$RL : RQ$

equal a given Ratio.

HENCE,

In the Diameter AB, of the given Circle ARB, there are given, the Points P, Q: to draw PR, QR, to R, a Point in the Periphery, such, that taking PL equal a given Line PI, the Ratio of RL to RQ shall be given.

THIS IS PROBLEM XVIII. B. II.

The Above is taken from the MATHEMATICAL REPOSITORY, Vol. I. New Series. See Question 9. and Solutions,

The same Method of Analysis will easily effect the Solution of the Problem, under the following more general Form.

When the Lines PR, QR, are to be drawn, so, that the Sum, or the Difference, of the Rectangles, under two given Lines PD, PF, and two Lines PE, QK, to which PR, QR, have given Ratios, shall be equal a given Space.

The following Enunciation is only a varied Form of Problem XXIII.

Given the Base, a Line, from the Vertex, to a given Point in the Base, and the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the Sides have given Ratios, to determine the Triangle.

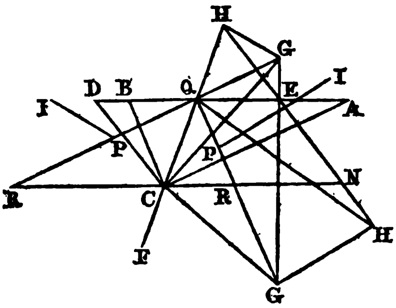
PROBLEM XXIV.

Given the Difference of the Angles at the Base, the Difference, or the Sum, of the Sides, and the Sum, or the Difference, of a Line, from the Vertex, making a given Angle with the Base, and a Line, to which the Difference of the Segments of the Base, by the Perpendicular, has a given Ratio, to determine the Triangle.

ANALYSIS.

From C, the Vertex of the required Triangle ABC, which we here suppose to be determined; apply, to AB the Base,

CQ = CB:



Let CG, bisecting the Angle ACQ, or its supplementary Angle ACF; and EG, bisecting AQ, at right Angles, meet each other at G.

The Triangle CBQ is isosceles,
Or, the Angle
CQB= CBQ:
Hence,
the Angle ABC—Angle BAC= Angle BQC—Angle BAC
equal Angle ACQ:
The Angle ACQ is, therefore, given.
Also, $AC \mp CB = AC \mp CQ$, is given;
and,
by a Property of plane Triangles, generally known,
$AC \mp CQ : CG$
is a given Ratio;
Or, CG is
a given Line.
Draw CD, making the given Angle with the Base;
also, EN, CN, parallel to DC, DA.
Let EH be the Line, to which the Difference of the Segments of the Base has the given Ratio; and draw GH, QH, QG.

Because of Parallels,

EN=DC,

and EH= the Line, to which the Difference of the Segments has the given Ratio;

then,

$DC \pm EH = EN \pm EH = HN$,

is a given Line.

Again, as

CB= CQ,

evidently, AQ equal the Difference of the Segments of AB, by a Perpendicular from C;

then,

AQ : EH, or EQ : EH,

is a given Ratio,

and, HEQ is a given Angle:

Consequently,

the Angle EHQ, and the Ratio EQ : QH, are given.

Also,

the Angle GQE=GCQ equal Half ACQ, is a given Angle, and GEQ is a right Angle;

Or,

EQ : EG

is a given Ratio:

That is,

the Ratio EG : EH, and the Angle EHG, are given.

Let GQ meet NC, at R; in GR, take GP= GC, and draw PI parallel to GH.

The Line HN, as well as the Angles NHG, NHQ, and HNR =CDB, are given;

Or, the Lines HG, HQ, NR, are given in position:

Also, GP is given,

and it makes a given Angle with GH, a Line given in position;

therefore,
a Line, parallel to GH, may be drawn, that shall pass
through P:
The position of IP, then, is given.
Moreover,
because of the equal Angles,
GQE, GRC, GCQ;
GQ : GC = GC : GR = QG : GP = GP : GR,
Or,
$GQ . GR = GP^2$.

WHEREFORE,

The four Lines HG, HQ, IP, and NR, are given in position: and, GR is to be drawn, making given Angles with those Lines, or parallel to a Line given in position, meeting HG, HQ, IP, NR, at G, Q, P, R, so, that $GQ . GR : GP^2$ shall be a Ratio of Equality.

This Problem, therefore, is A PARTICULAR CASE OF THE XX. B. II.

The Analysis will differ very little from the above, when our Problem has the following more general Enunciation.

Given the Difference of the Angles at the Base, a Line, to which the Sum, or the Difference, of the Sides, has a given Ratio, and a Line having a given Ratio to the Sum, or to the Difference, of two Lines, one of which has a given Ratio to a

Line drawn from the Vertex, making a given Angle with the Base, and the other, a given Ratio to the Difference of the Segments of the Base, by the Perpendicular.

PROBLEM XXV.

Given in position, the right Lines CL, CR, and the Points N, P, Q: to draw the Lines NL, NR, including a given Angle, and meeting CL, CR, at L, R, so, that right Lines drawn to the given points, P, Q, from the points L, R, shall contain a given Angle.

ANALYSIS.

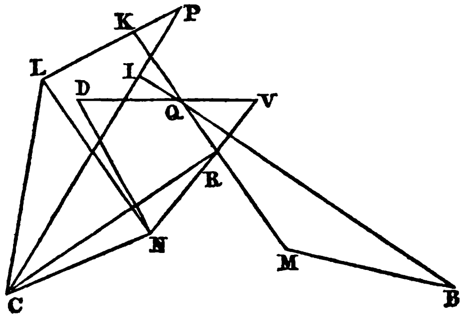
The Lines NL, NR, LP, RQ, being supposed drawn; join the Points N, C; and P, C.

Also, make CND, PIQ, equal to the given Angles.

The Lines NC, PC; also, the points N, Q; are given in position; and,

CND, PIQ,

are given Angles:



Hence, ND and IQ, are given in position.
Make ND=NC, and, in IQ, take QB=PC:
Then,
the points D and B are given.
Draw DV, BM, making the Angle NDV=NCL,
QBM=PCL; and, let QR, NR, meet LP, BM, DV, at K,
M, V.
The Lines DN, BQ, are given in position; D, B, are given
points, and NDV, QBM, are given Angles;
therefore,
DV, BM,
are given in position.
Also, since the Angle
CND=LNR, NDV=NCL, QIP=QKP, and
QBM=PCL; it evidently follows, that the Angle
DNV=CNL, and IQK=IPK=BQM;
Or, the Triangles
NCL, NDV; and PCL, BQM;
are equiangular, and mutually equal:

Having
NC : ND=CL : DV ; and, PC : QB=CL : BM :
Also,
ND=NC, and QB=PC :
Consequently,
DV=CL=BM.

The Points N, Q, and the right Lines CR, DV, BM, then, are given in position : to draw NR, QR, to R, in the Line CR, so, that meeting DV, BM, at V, M ; the Segments DV, BM, cut off, shall be equal to each other.

This is A PARTICULAR CASE OF PROBLEM, XXVI. B. I.

* * *

Problem XXV. may be enunciated as follows.

Given two opposite Angles, to determine the Trapezium ; when the Sides, containing one of the given Angles, pass through two given points, those containing the other given Angle, intersect at another given Point, and, the two remaining angular Points are posited upon two right Lines, given in position.

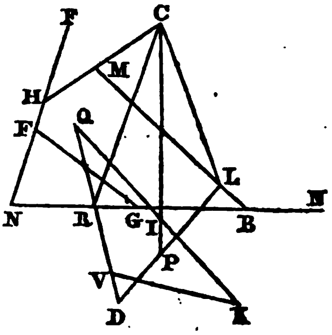
Among the Deductions, appear the Problems below.

The Lines DR, KNM, and the Points P, Q, are given in position : to determine the point R, in DR, such, that drawing RP, RQ ; also, PM, QN, making RPM, RQN, equal to two given Angles; the Ratio, Sum, or Difference, of the Segments KM, KN, shall be given.

Given in position, two Points, and three right Lines: to assign the position of a Point, in one of the Lines given in position, such, that, thence, drawing Lines to the two given Points, and, from those points, Lines making given Angles with the Lines first drawn; the Lines, last drawn, shall meet the two remaining Lines given in position, and cut therefrom, two Segments, of which, the Ratio, Sum, or Difference, shall be given.

PROBLEM XXVI.

The Points M, N, P, Q, are given in position, as also the right Lines CL, CR: to draw the Lines ML, NR, including a given Angle, and meeting the Lines CL, CR, at L, R, so, that drawing LP, RQ, to the given points P, Q; the Lines LP, RQ, shall include a given Angle.



ANALYSIS.

Suppose the position of ML, NR, to be determined.
Draw LP, RQ, MC, PC: let ML, NR, intersect at B, and
LP, QR, at D.
From N, Q, draw NH, QI, making, with CM, CP, the Angles
CHF, CIQ, equal to the given ones MBN, PDQ.
Take, in NH and QI; NF=MC, and QK=PC.
The point N, and the Line CM,
are given in position;
and,
the Angle CHF, and the Line CM,
are given;
NH or NF,
is, thence, given in position,
and, also, F is a given Point.

Again,
the position of the Point Q, and of the Line CP,
is given;
also,
the Angle QIC, and the Line CP are given:
Then,
the position of QI, or QK, and the point K, are given.
Draw FG, KV, making the Angle NFG=MCL, and
QKV=PCL:
The Intersections of NR, FG; and of QR, KV; being at G,
and V.
Since FN, KQ, are given in position; F, K, given Points,
and NFG, QKV, given Angles;
it follows, that the Lines
FG, KV,
are given in position.
Moreover,
the Angle MHF=MBN, and QIC=QDP,
Or, the Points
N, H, M, B; and Q, I, P, D;
are in the Periphery of a Circle:
Consequently, the Angle
FNG=CML, and KQV=CPL:
That is, the Triangles
FNG, CML; and QKV, CPL;
having, also, the Angle
NFG=MCL, and QKV=PCL;
NF=MC, and QK=PC;
are equiangular, and mutually equal:
Or,
FG=CL=KV.

We have then, Given in position, the Lines CR, FG, KV, and the two Points N, Q: to inflect the Lines NR, QR, to a point R, in CR, such, that NR, QR, shall cut off, from FG, KV, the equal Segments FG, KV.

A PARTICULAR CASE OF THE XXVI. PROBLEM, B. I.

◆

The preceding Problem might have been proposed under the following Form.

Given two opposite Angles, to construct the Quadrilateral, so, that the Sides shall pass through four given Points, and the two remaining angular points be posited on two right Lines, given in position.

Two Deductions are presented for Determination.

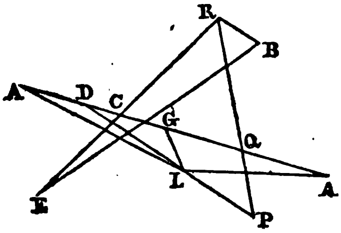
Given in position, the three Points P, Q, R, and the two Lines DV, HKL: to draw PV, QV, to intersect at V, in DV, so, that drawing PL, RK, making given Angles with VP, VQ, meeting HL, at L, K; the Ratio of the Segments HL, HK: or the Sum, or Difference, of Lines, to which HL, HK, have given Ratios; shall be given.

Three Points P, Q, R, and three right Lines DV, HL, MK, are given in position: to inflect PV, QV, to V, in DV, so, that drawing PL, RK, making given Angles with VP, VQ, meet-

ing HL, MK, at L, K; the Ratio of the Segments HL, MK: or the Sum, or Difference, of two Lines, to which HL, MK, have given Ratios; shall be given.

PROBLEM XXVII.

The Lines DQ, ER, and the point P, are given in position: to draw the right Line PQR, meeting DQ, ER, at Q, R, so, that the Sum, or the Difference, of Lines, to which DQ, ER, have given Ratios, shall be given.



ANALYSIS.

The position of the Line PQR, we will suppose to be determined.

Let DG, GA, be the Lines to which DQ, ER, have the given Ratios.

Then DG$\pm$ GA= DA, is a given Line,
and, as D is a given point,
the point A is, therefore, given.

Join P, D; draw GL parallel to RP, meeting PD at L, and draw AL.

By parallel Lines,
DP : DL=DQ : DG,
is a given Ratio,
and DP is given in position and Length;
Or,
the point L is given;
and, consequently, AL, joining the given Points
A, L,
is given in Length and position.

Draw EB, RB, making the Angle REB=GAL, and ERB= AGL.

The Angle GAL, and its Equal, CEB, are given, and, E is a given Point;
hence, EB is given in position.

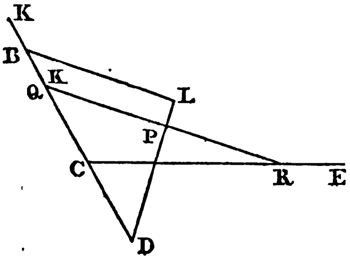
Also, the Triangles
AGL, ERB,
having two Angles, in each, equal, have, likewise, their remaining Angles equal;
Or, they are equiangular Triangles:
That is,
LA : BE= GA : RE,
is a given Ratio:

It follows, that EB is given, B, thence, a given Point, and the Angle
AGL =ERB= CQP.

We have therefore given, The Points P, B,

and the Lines CQ, CR, in position: to draw PR, BR, to R, in CR, making equal Angles with CQ, CR.

WHICH IS OUR GENERAL PROBLEM.



OTHERWISE.

Let DB, BK, be the Lines to which DQ, ER, have the given Ratios:

Then, DK, or the Point K, is given.

Draw BL parallel to QP, meeting a Line through D and P, at L.

Because of parallel Lines,

DP : DL = DQ : DB

equal a given Ratio,

and DP given,

in Length and position:

Hence, DL, or the Point L, is given.

Now,

PR is parallel to LB, the Points P, L, E, K, are given, and

ER : BK

is a given Ratio.

THEREFORE,

Our Problem is, To draw, from the given Points P, L, the Parallels PR, LB, meeting EC, KC, two Lines given in position, at R, B, and determining the Segments ER, KB, in a given Ratio.

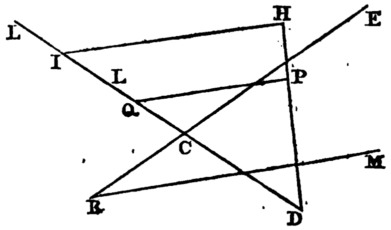
A CASE OF PROBLEM XI. B. I.

The following deduced Problem is added.

The Lines AD, AB, and PM, parallel to AB; also, the Points P, M, Q; are given in position: to draw the Parallels PB, AL, meeting AB, MP, at B, L, so, that drawing BD parallel to another Line given in position; and making MK : ML equal a given Ratio: the Points D, K, Q, shall be in the same right Line.

PROBLEM XXVIII.

The right Lines DC, EC, and the points P, M, are given in position: to draw the Parallels PQ, MR, meeting the Lines CD, CE, in Q, R, so, that the Sum, or the Difference, of Lines, to which the Segments DQ, ER, have given Ratios, shall be given: or, that the Rectangle of DQ, ER, shall be given.



ANALYSIS.

Suppose the required Parallels to be drawn.

Take DI, IL, the Lines to which DQ, ER, have the given Ratios.

Then, $DI \pm IL = DL$, and the Point D, being given; L is, consequently, a given Point.

Join D, P; and draw IH parallel to QP.

Because of Parallels,

$DP : DH = DQ : DI$,

is a given Ratio,

and DP given;

hence,

DH, or the Point H, is also given.

Wherefore,

Since the points H, M; as also, the Ratio IL : ER;

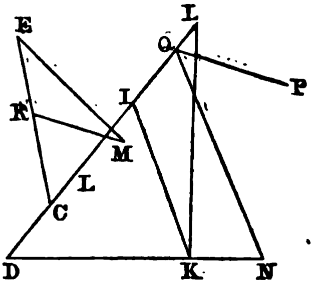
are given,

and HI parallel to PQ, or to MR;

The Cases for the Sum and Difference, are, obviously, the same as the XI. Problem, B. I.

THAT IS,

We are required to draw, from the two given Points H, M, the Parallels HI, MR, meeting the Lines LI, ER, given in position, at I, R, so, that the Segments LI, ER, cut off, shall have to each other, a given Ratio.



OTHERWISE.

Let DI, IL, be the Lines to which DQ, ER, have the given Ratios:

Then, DL, and the Point L, are given.

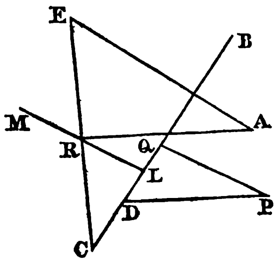
Draw EM, LK, IK, making the Angle ILK= REM, and LIK=ERM.

The Point L, and the Angle CLK= CEM, are given, or, LK is given in position.

Also, the Triangles REM, ILK, are equiangular,
having
EM : LK=ER : LI,
a given Ratio,
and EM given;
hence, LK, or K, is given.

Draw QN parallel to IK, meeting a Line through D, K, at N.
Now,
DN : DK=DQ : DI,
is a given Ratio,
and, DK is given in Length and position;
Or, the point N is given:
Also,
the Angle LKI=EMR, and NQ, PQ, are parallel to KI, MR:
Or,
NQ, PQ, make equal Angles with LK, EM,
and intersect at Q, in the Line DL:

A CASE OF THE GENERAL PROBLEM.



When the RECTANGLE of DQ, ER, is given.

Join P, D; draw EA, RA, making the Angle CEA=PDQ,
and ERA=PQD.
The Lines CD, DP, are given in position,
Or,
the Angle PDQ, or CEA, is given:

Also,
CE is given in position, and E is a given Point,
the position of EA, then, is given.
Now,
the Triangles PDQ, AER, are, evidently, equiangular, where
PD : DQ = RE : EA :
That is,
PD . EA = DQ . ER,
is a given Rectangle,
and the Line PD is given :
Therefore,
EA, and the Point A, are given :
Also, the Angle
MLB = PQD = ARE :
The Intersection of CD, MR, being at L.

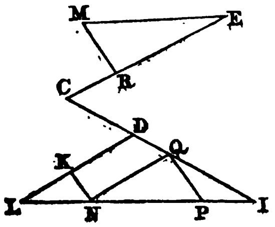
CONSEQUENTLY, IN THIS CASE,

We have to draw, from the given Points A, M, the Lines AR, MR, to R, in CE, so, that AR, MR, shall make equal Angles ARE, MLB, with the Lines CE, CB, given in position.

WHICH IS THE GENERAL PROBLEM.

OTHERWISE.

Draw ME, and PI parallel thereto, meeting DC, and DL, a parallel to CE ; at I and L.
Then DL, IL, and the Points I, L, are given in position.
Draw QN, NK, parallel to DL, QP, meeting IL, DL, at N, K.



The Rectangle DQ . ER, and the Ratio DQ : NL= DI : IL,
are given;
Or, the
Rectangle NL . ER is given:
Because of similar Triangles,
EM : ER =LN : LK,
then,
EM . LK= NL . ER,
is a given Rectangle,
and EM a given Line:
Therefore, LK, or K, is given:
Also, because of Parallels,
IQ : IN= ID : IL,
is a given Ratio,
And PQ, KN, are parallel.

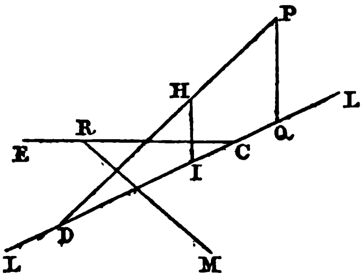
HENCE,

The Points P, K, and the Lines IQ, IN, are given in position: to draw the Parallels PQ, KN, meeting the Lines given in position at Q, N, so, that the Ratio IQ : IN shall be given.

A PARTICULAR CASE OF THE XI. B. I.

PROBLEM XXIX.

The Lines DC, EC, and the points P, M, are given in position: to draw the right Lines PQ, MR, including a given Angle, and meeting CD, CE, in Q, R, so, that the Sum, or the Difference, of Lines, to which the Segments DQ, ER, have given Ratios, shall be given: or, that the Rectangle of DQ, ER, may be given.



ANALYSIS.

Admit the Position of PQ and MR to be assigned: and let DI, IL, be the Lines, of which the Sum, or the Difference, is given.

Then, DI$\pm$IL =DL, is a given Line,
and D is a given Point;
therefore, the Point L is given.
Draw DP, also IH, parallel to QP, meeting DP at H.

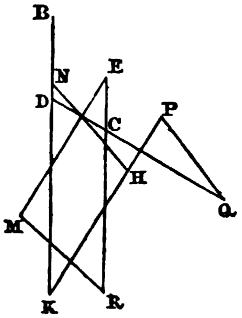
Now,
DQ : DI
is a given Ratio,
and, obviously,
DP : DH = DQ : DI;
then,
DP : DH, or DH : HP,
is a given Ratio:
Or, the given Line DP is divided in a given Ratio, at H:
That is, H is a given Point.
Hence, from the given Point H, the Line HI is drawn, parallel
to PQ; and PQ makes a given Angle with MR;
or, HI makes a given Angle with MR:
And, ER : LI equal
a given Ratio.

THAT IS, MR, HI, are to be drawn, from the given Points M, H, to include a given Angle, and to cut off, from the Lines CE, CL, given in position, the Segments ER, LI, which shall have, to each other, a given Ratio.

WHICH IS THE FORM OF PROBLEM XVII. B. I.

OTHERWISE.

Parallel to the Line, passing through E, M, draw PK, meeting
DK, parallel to EC, at K.
The points P, D, are given,
and,
PK, DK,
are parallel to Lines given in position,
Or PK, DK, and the point K,
are given in position.

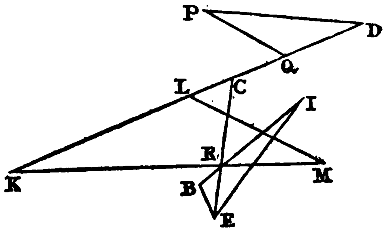


Let BN, NK, be the Lines, to which DQ, ER, have the given
Ratios, and draw NH parallel to MR.
Then, KB and the point B, are given:
Also, because of Parallels,
ER : KN = EM : KH
equal a given Ratio,
and EM is given:
Therefore, KH, and the point H, are given.
Now,
HN is parallel to MR,
Or,
PQ, HN,
include the given Angle, and determine the Segments
DQ, BN,
in a given Ratio.

THAT IS,

The Points P, H, and the Lines DQ, BN, are given in position: to draw PQ, HN, including a given Angle, and determining, on the Lines given in position, the Segments DQ, BN; having a given Ratio.

THIS IS THE XVII. B. I.



When the RECTANGLE of the Segments DQ, ER, is given.

Draw PD, EB, BR, making the Angle CEB= CDP, and ERB= DPQ.
Now, DQ . ER is a
given Rectangle,
and the Triangles PDQ, REB, being equiangular,
PD : DQ= RE : EB,
Or,
PD . EB= DQ . ER;
then, PD . EB is a given Rectangle, and PD a given Line;
therefore, EB is a given Line:
The Position of EB is, also, given: For, CE is given in position, and CEB is a given Angle:
Then, the Point B is given.
Draw ML, EI, making ML parallel to PQ, and the Angle BEI=RML equal the given Angle; and, let ML, MR; BR, meet DC; EI, at L, K; I.

The Line BE is given in position,
and, BEI is a given Angle;
Or, the
Position of EI is given.
Also, the Angle
KML= BEI,
and, by Parallels,
KLM= DQP= IBE:
Consequently, the Angle
LKM =BIE.

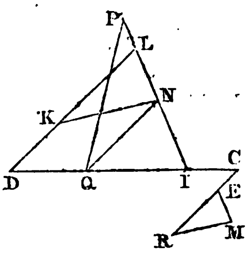
THAT IS,

The Lines EC, DC, EI, and the points M, B, are given in position: and our Problem is, to draw MR, BR, intersecting at R, in EC, and making, with DC, EI, the equal Angles CKM, BIE.

THIS IS A CASE OF THE GENERAL PROBLEM.

OTHERWISE.

Join the Points E, M.
Draw PI parallel to EM; also, DL, QN, parallel to EC; and NK parallel to MR.
The Points P, D, and the Lines EM, EC, CD, are given in position,
therefore, the Position and Length, of
PI, DI, IL,
are given.



Also, DQ . ER is a given Rectangle, and DQ : LN equal DI : LI, is a given Ratio: Then, LN . ER is a given Rectangle.

Now, because of parallel Lines, we have EM : ER = LN : LK, Or, EM . LK = LN . ER, is a given Rectangle, and, EM is a given Line: Then, LK, or the point K, is given.

Moreover, KN is parallel to RM, and, the Lines PQ, RM, include a given Angle; therefore, the Lines PQ, KN, include a given Angle: Also, PQ, KN, are drawn from the given Points P, K, and determine IQ : IN = ID : IL, a given Ratio.

WHENCE,

We are here required to draw, from the given Points P, K; the Lines PQ, KN, including a given Angle, and determining, on the Lines ID, IL, the Segments IQ, IN ; or DQ, LN: having a given Ratio.

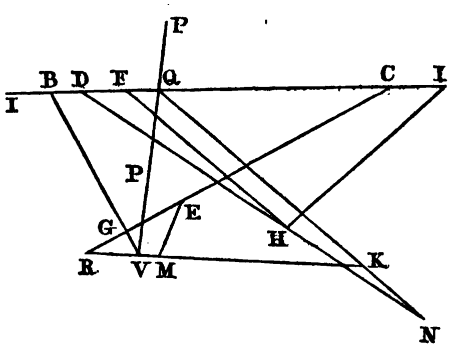
A PARTICULAR CASE OF THE XVII. B. I.

PROBLEM XXX.

The Lines BG, BD, EG, and the Points P, M, are given in position ; as also, the Points D, E, in the right Lines BD, GE: to draw the Lines PV, MV, having their Point of concourse V, in the Line BG, and meeting BD, EG, at Q, R, so, that the Sum, or the Difference, of Lines, to which the Segments DQ, ER, have given Ratios, shall be given : or so, that the Rectangle of the Segments DQ, ER, shall be equal a given Magnitude.

ANALYSIS.

We will suppose the position of PQ, MR, to be determined: and FD, FI, the Lines to which DQ, ER, have the given Ratios.



Draw EM, IH, FH, making the Angle FIH=GEM, and IFH=GRM.

Then, FI$\pm$FD, or DI, and the point D, are given; or, I is a given Point.

Also,

since FIH=REM equal a given Angle, and the point I given; the position of IH is given.

And, as

FIH, REM,

are, obviously, equiangular Triangles,

having EM : IH=ER : IF, a given Ratio,

and EM given:

Therefore, IH, or the point H, is given.

Join D, H; and, let QKN, drawn parallel to FH, meet RM, DH, at K, N.

The Ratio DQ : DF=DN : DH, is given,

and, DH is given in Length and Position,

hence, DN, and, therefore, the point N, are given.

Moreover,
the Angle CRK=CFH =CQK, because of Parallels;
Or, the Points C, Q, R, K, are in a Circle;
and, consequently, the Angle
QKR =QCR:
That is,
NQ, MV,
contain a given Angle.

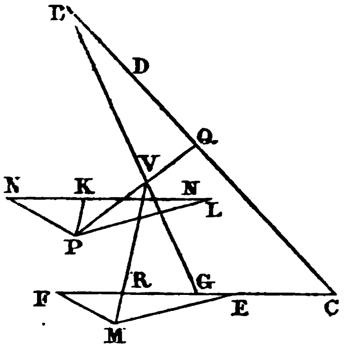
WHEREFORE,

The Lines CQ, BV, and the Points N, M, P, are given in position: and, we have to draw NQ, MV, including a given Angle, and meeting the Lines given in position at Q, V, so, that the Line QV shall pass through the remaining point P.

THIS IS THE XXV. PROBLEM, B. I.

OTHERWISE.

Draw ME, and PL parallel thereto:
Make ME : PL equal the second given Ratio.
Then,
PL
is given in position and Length,
Or, L is
a given Point.
Draw LN parallel to CE; and, let NK, KL, be the Lines, to
which DQ, ER, have the given Ratios.
Join K, P.



Now, LN is given, in position and Length, or, the point N is given:
Also, since
ME : PL=ER : LK,
equal the second given Ratio,
and,
PL, LK, parallel to ME, ER:
Therefore,
PK is parallel to the Line MR.

THAT IS,

The points P, M, and the Lines BG, BD, NL, are given in position: to draw PV, MV, to V, in BG, such, that PV, and PK, a Parallel to MV, meeting BD, NL, at Q, K; the Ratio of the Segments DQ, NK, shall be given.

A CASE OF PROBLEM IX. B. II.

OR THUS.

Draw PN; and MF, parallel to PN, meeting CE at F.

Then,

DQ : NK,

and,

FR : NK= FM : NP;

are given Ratios:

Hence, DQ : FR is a given Ratio.

THEREFORE,

We have only to draw PV, MV, to V, in BG, determining the Segments DQ, FR, in the Lines CB, CF, having a given Ratio.

WHICH IS PROBLEM XXVI. B. I.

When the RECTANGLE of the Segments DQ, ER, is equal a given Magnitude.

Draw EM, DA, QA, making the Angle CDA= GEM, and BQA =EMR.

The Rectangle DQ . ER is given:

And, because of the equiangular Triangles ADQ and REM,

DA : DQ= ER : EM,

Or,

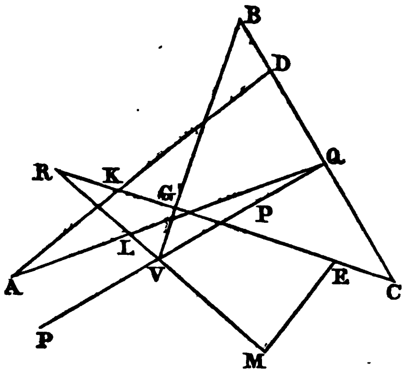
DA . EM=DQ . ER,

a given Rectangle,

EM a given Line, and CDA=GEM, a given Angle:

Hence, DA is given, in Length and position;

and, therefore, A is a given Point.

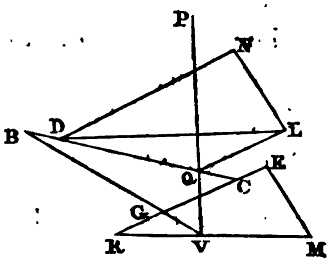


Let DA, QA, intersect CG, MR, at K and L.
The Lines CG, DA, are given in position,
then,
K is a given point, or AKR is a given Angle:
Also, the Angle KAL=KRL; or, the Angle ALR=AKR;
Or,
the Angle ALR is given:
That is,
AQ, MV,
include a given Angle.

Therefore,

The Points A, M, P, and the Lines CQ, BV, are given in position: to assign the position of AQ, MV, including a given Angle, so, that meeting the Lines given in position, at Q, V; the three Points P, Q, V, shall be in the same right Line.

Which is Problem XXV. B. I.



Otherwise.

Join the points E, M.

Draw DN, QL, parallel to CE; and DL, LN, parallel to RM, ME; meeting QL, DN, at L, N.

The Rectangle DQ . ER, and the Ratio DQ : NL, are given; then,

NL . ER

is a given Rectangle.

Now, the Triangles DNL, REM, are, obviously, similar, having

DN : NL = RE : EM,

Or,

DN . EM = NL . ER,

a given Rectangle;

and, the Line EM is given:

Then, DN is given.

The Line DN is given, also, in position, or, the point N is given; and, therefore, NL, being parallel to EM, is given in position.

Consequently,

We have given in position, The Lines BG, DC, NL, and the Points M, P, D: to draw MV, PV, to V, in BG, such, that PV, and DL, a Parallel to MV, meeting DC, NL, at Q, L; the Ratio DQ : NL, may be given.

A Case of the IX. B. II.

Or, of the XXVI. B. I.
as in the Case for the Sum and Difference.

From the XXVIII., and the two following Problems, we have purposely omitted making the usual *Deductions*: leaving the Notice of those, which the Course of the Solutions may have suggested, as an Exercise, essential to the Geometer.

With Problem XXX., we close the Examples in Analysis.

Book III., or our intellectual Gymnasium, opens with an Arrangement of the Problems that have been deduced, during the preceding Inquiries.

To extend and improve these Geometrical Exercises, we have given an additional Series, of selected, and original Problems.

BOOK III.

GEOMETRICAL

EXERCISES.

DEDUCTIONS,

FROM BOOK I.

PROBLEM I.

To determine two Lines, of which, the Rectangle and Ratio are given.

II.

To draw, from two given Points, two Lines intersecting each other in a right Line given in position, and, with two other Lines, also given in position, making Angles, of which, the Sum, or the Difference, shall be given.

III.

To determine two Lines, having to each other a given Ratio, such, that adding a given Line to each, the Rectangle of the compound Lines, shall be equal to a given Square.

IV.

To determine, in a right Line ACP, given in position, and intersecting a given Circle in C, P; a Point M, such, that drawing a Tangent to the Circle, from M; the Segment AM shall be equal to that Tangent; or shall have any given Ratio to it.

V.

The point N, in the Line AP, is given: to determine another point M, in AP, such, that the Rectangle under the Segment MP, and a given Line, shall be equal, or have any given Ratio, to the Square of the Tangent, from M, to any Circle, passing through the points N, A.

VI.

To draw, from a given Point, a right Line, meeting two right Lines given in position, and cutting off Segments, such, that the Sum, or the Difference, of one of the Segments, and a Line to which the other Segment has a given Ratio, shall be given.

VII.

Given the Perimeter, and two Lines having given Ratios to the Perpendicular, and the Line bisecting the vertical Angle; to determine the Triangle.

VIII.

Given the Base, and the Difference of the Angles at the Base, to determine the Triangle, when the vertex is posited in a right Line given in position.

IX.

Given in position, a Point and three right Lines: to draw, from the given point, two right Lines, making, with one of the right Lines, two Angles, of which, the Difference, or the Sum, shall be given, and cutting off, from the two remaining Lines, Segments, having a given Ratio.

X.

Three right Lines and a Point, being given in position: to draw, from the given point, two right Lines, making, with one of the Lines given in position, equal Angles, and cutting off, from the two remaining Lines, Segments, of which, the Ratio, the Sum, or the Difference, shall be given.

XI.

From two given Points, to draw two right Lines, making equal Angles, with a right Line given in position, and cutting off Segments, from two other right Lines, given in position, such, that the Ratio, Sum, or Difference, of those Segments, may be given.

XII.

Through a given Point, to draw a right Line, constituting, by its Intersection with two Lines given in position, a Triangle of a given Magnitude.

XIII.

Three right Lines and a Point, are given in position : to draw a Line, through the given point, intersecting two of the given Lines, in two Points, such, that the Sum, or the Difference, of the Perpendiculars, from the Points of intersection, upon the remaining Line, shall be given.

XIV.

Given the Hypothenuse, and the Line bisecting the Right Angle, to determine the Triangle.

XV.

✓ To determine the right angled Triangle, there are given, one of the Sides, and the Perpendicular, from the right Angle, upon the Line bisecting the Angle opposite the given Side.

XVI.

From the Point A, in the Line AD, to draw AE, to E, a point in the perpendicular DE, such, that the right Line bisecting the Angle AED, shall touch a given Circle, to the Centre D.

XVII.

In the Line DH, perpendicular to the Radius DG, of the given Circle DGI, is the point A: to draw the Tangent PI, touching the Circle at I, and meeting DG at P, so, that joining the Points I, A, and drawing PH parallel to IA, meeting DA at H; the Segments AD, AH, shall be equal.

XVIII.

The Line AD is perpendicular to the Radius DG, of the given Circle DIG: to draw AIE, meeting the Circle at I, and DG at E, so, that EI may be equal to ED.

XIX.

In a right Line, intersecting two Circles given in position and Magnitude, to determine a Point, such, that Tangents to the Circles, from the Point so determined, shall have, to each other, a given Ratio.

XX.

Given the two Sides, and the Sum, or the Difference, of two Lines, to which the Segments of the Base, by a Perpendicular from the vertical Angle, have given Ratios, to determine the Triangle.

XXI.

To determine a Point P, in the Line passing through the Centres, of two given Circles, such, that drawing PQR at right Angles to the Diameters, meeting the Circles at Q, R; the Ratio of PQ to PR shall be given.

XXII.

To add, to a given Line, or to take from it, another Line, such, that the Difference between a given Square, and the Square of the compound Line, or the Square of the remaining Line; shall have, to the Difference between another given Square, and the Square of the Line added to, or taken from, the given Line, a given Ratio.

XXIII.

To take from, or to add to, a given Line, another Line, such, that the Rectangle under the Remainder, or the Sum, and another given Line, shall be equal to the Square of the Line taken from, or added to, the given Line.

XXIV.

Given two Lines, to determine another Line, such, that adding it to one of the given Lines, and taking it from the other; the Rectangle under the Sum and the Part added, shall be equal to the Square of the Difference.

XXV.

The Line FD is perpendicular to the given Line AFI: to draw AD, to a Point D, in FD, such, that making ADB a right Angle, IB shall be equal to BD.

The Point B being the Intersection of DB, AI.

XXVI.

From the Centre A, of the given Circle AIN, to deflect the right Lines AN, AD, including a right Angle, meeting the Circle at N, and a Perpendicular FD, to the Radius AI, at D, so, that the Points N, D, I, shall be in the same right Line.

XXVII.

To describe a Circle that shall have its Centre in the right Line IA, pass through I, and intersect the right Line FD, given in position, in D, so, that DA being drawn, shall be a Tangent to the Circle.

XXVIII.

The right Lines BM, DN, and the Point L, are given in position: to draw LM, MN, meeting BM, DN, at M, N, making the Angle LMN of a given Magnitude, so, that the Ratio of BM to DN shall be given.

XXIX.

Given in position, the Points L, V, and the right Lines BL, BM, DV, DN: to draw LM, VN, making equal Angles with LB, VD, and meeting BM, DN, at M, N, so, that the right Line passing through the Points M, N, shall make a given Angle with LM, or VN.

XXX.

In each of two right Lines, given in position, to determine a Point, such, that a Line, passing through those Points, shall be parallel to another Line, given in position, and, that a Circle, to contain a given Angle, being described through the same Points, shall pass through a given Point.

XXXI.

Two right Lines and a Point, being given in position: to describe, through the given Point, a Circle, such, that the Segments of the Lines, intercepted between their Point of concourse, or between given Points in the Lines, and the Intersections of the Circle, shall have a given Ratio to each other.

XXXII.

Two Points and three right Lines, being given in position: to draw a right Line, making a given Angle with one of the Lines given in position, and meeting the remaining ones, so, that Lines, drawn from the Points of Intersection, to the given Points, shall either include a given Angle, or have their Point of concourse, in another right Line, given in position.

XXXIII.

To construct the Quadrilateral, when one Side is given in Length and position, the two Sides adjacent to the given one, are parallel, the remaining Side passes through a given Point, and the angular Points, at its Extremities, are posited on two right Lines given in position.

XXXIV.

The Point R, and the right Lines AL, MV, EF, are given in position; of which, MN, EF, are parallel: to inflect MV, NV, to a Point V, in EF, such, that making MI : IV= NH : HV, equal a given Ratio, and drawing RA, RL, through I, H: the right Lines passing through the Points M, A; and N, L; shall be parallel.

XXXV.

Given the Diagonals, the Angle they make with each other, and the Position of one of them, to determine the Quadrilateral, when either Pair of the opposite Sides, has its Point of concourse given in position.

XXXVI.

Given the Diagonals, their point of Intersection, the Angle they include, and the position of one of them, to constitute the Trapezium; when either Pair of the opposite Sides produced, intersects each other, in a right Line given in position.

XXXVII.

Given in position, four right Lines and a Point: to draw, from the given Point, two right Lines, that shall intercept, on one of the Lines given in position, a Segment of a given Length, and meet two of the remaining Lines given in position, so, that a Line, passing through the Points of Intersection, shall be parallel to the fourth Line, given in position.

XXXVIII.

One of the angular Points of a Triangle is given in position, also, the Magnitude of that Angle, to determine the Triangle; when the other Angles are posited upon two Lines given in position, and the Side, opposite the given Angle, passes through a given Point.

XXXIX.

To describe a Triangle, similar to a given Triangle, having one of its angular Points given in position, and the remaining Angles posited upon two right Lines, given in position.

XL.

To determine a Point, in a Line given in position, such, that drawing, thence, a Line to a given Point, also, a Line making a given Angle, with that drawn to the given Point, and a Line parallel to another, given in position; the two latter shall cut off, from two other Lines given in position, Segments, from given Points therein, of which, the Ratio, Sum, or Difference; or the Sum, or Difference, of two Lines, to which those Segments have given Ratios: shall be given.

XLI.

Given one of the Angles, to determine the Triangle; when the Sides pass through three given

Points, and the remaining angular Points are posited upon two right Lines, given in position.

XLII.

To determine the Quadrilateral, one of the Sides is given, in Length and Position, the Side, opposite the given one, passes through a given Point, the angular Points, at its Extremities, are posited on two Lines given in position, and the remaining Sides include a given Angle.

XLIII.

Two Points, and two right Lines, are given in position: to draw, from one of the given points, two Lines, including a given Angle, meeting the Lines given in position, so, that Lines, drawn from the Intersections, to the remaining given point, shall include a given Angle.

XLIV.

Three Points and two right Lines, being given in position: to draw, from one of the given points, two Lines, including a given Angle, and meeting the Lines given in position, so, that

Lines, from the Points of Intersection, to the two remaining points given in position, shall be parallel.

XLV.

To draw, from a given Point, two Lines, intercepting, on a Line given in position, a Segment of a given Length, and cutting off, from two other Lines given in position, between the Points of Intersection and given Points in those Lines, two Segments, having a given Ratio.

XLVI.

Given in position, four right Lines, and two Points: to draw, from one of the given points, two Lines, meeting two of the Lines given in position, at P, Q, and cutting therefrom, Segments having a given Ratio, Sum, or Difference, so, that Lines drawn, from the remaining given point, to the Points P, Q, shall cut off, from the two remaining Lines given in position, Segments, of which the Ratio, Sum, or Difference, shall also be given.

XLVII.

Four right Lines, and three Points, are given in position: to draw, through one of the given points, a Line, meeting two of the Lines given in position, so, that Lines, drawn from the points of Intersection, to the two remaining Points given in position, shall make equal Angles with the two remaining Lines, given in position.

DEDUCTIONS,

FROM BOOK II.

PROBLEM XLVIII.

The Diameter AB, of a given Circle, a Point P in that Diameter, and the right Line MR, are given in position: to determine the Point R, in MR, such, that PQ, having a given Ratio to MR, being applied to the Circle, the Line passing through Q, R, shall be at right Angles to AB.

XLIX.

The Line DR and the Parallels PQ, KL, are given in position: to draw PR, QR, to intersect at a point R, in DR, so, that meeting KL, at the Points M, N; the Sum, or the Difference, of KN, and a Mean-proportional between KL, KM, shall be equal to a given Line.

L.

To divide a given circular Arc, into two Parts, such, that the Sum, or the Difference, of two Lines, to which the Tangents of those Parts have given Ratios, shall be given.

LI.

Given the vertical Angle, the Length of a Line, from that Angle, making a given Angle with the Base, and the Sum, or the Difference, of two Lines, to which the Segments of the Base, by the Line from the Vertex, have given Ratios, to determine the Triangle.

LII.

To determine the Triangle, given the Hypothenuse, and the Sum, or the Difference, of two Lines, to which one Side, and the alternate Segment of the Hypothenuse, by a Perpendicular from the right Angle, have given Ratios.

LIII.

The Line GR is perpendicular to the Diameter BD of a given Circle, and P is a given Point in

BD, or in BD produced: to draw PQ, to a Point Q, in the Periphery, such, that drawing QR, making a given Angle with GR, and adding to, or taking from, PQ, QR, the given Lines PI, QV; or adding PI to PQ, and taking QV from QR: the Ratio IQ : QV shall be given.

LIV.

The right Lines VD, LE, IF, and the Point P, are given in position: to draw PF, PT, meeting IF, LE, VD, at F, T, E, D; so, that FT : ET shall be a given Ratio, and the Sum, or the Difference, of two Lines, to which LE, VD, have given Ratios, equal a given Line.

LV.

Through two given Points, to describe a Circle, intersecting a right Line given in position, in two points; such, that Chords, drawn through the two points of Intersection, and through two other given Points; shall intersect another right Line given in position, in the same Point.

LVI.

The Lines BD, BE, QR, and the Point P, are given in position; also, the Points Q, R, in the

the given ones; so, that one of the Chords shall pass through a given Point, another shall make a given Angle with that passing through the given point; and the remaining Chord shall cut off, from two Lines given in position, two Segments, making the Sum, or the Difference, of two Lines, to which those Segments have given Ratios, equal a given Line.

LXIII.

To determine the Triangle, there are given, the Sum, or the Difference, of two Lines, to which the Base, and a Line, drawn from the Vertex, making a given Angle with the Base, have given Ratios; also, two Lines, to which the Diameter of the inscribed Circle, and the Difference between the Sum of the Sides and the Base, have given Ratios.

LXIV.

Having a Point and a right Line, given in position: to draw two Lines, from the given point, including a given Angle, that shall cut off, from the Line given in position, two Segments, between the points of Intersection, and a given Point, or given Points, in that Line, the Rectangle of which shall be given.

LXV.

Two given Circles, a Diameter AB, of one of them, and a Chord KL, of the other, are given in position: to assign, in AB, the points P, Q, such, that AP : PB = AQ : QB, and, that PK, QL, shall intersect in the Periphery of the Circle KL.

LXVI.

Given in position, four right Lines: to assign the position of a Point, such, that drawing, from that Point, four right Lines, meeting those given in position, in given Angles; the Sum, or the Difference, of two Lines, to which two of the Lines so drawn, have given Ratios, shall be given: and, the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the two remaining Lines, drawn from the point assigned, have given Ratios, equal a given Space.

LXVII.

To draw, from the Points P, Q, posited in the Diameter AB, of a given Circle, the Lines PR, QR, to R, a Point in the Periphery of the given the given ones; so, that one of the Chords shall pass through a given Point, another shall make a given Angle with that passing through the given point, and the remaining Chord shall cut off, from two Lines given in position, two Segments, making the Sum, or the Difference, of two Lines, to which those Segments have given Ratios, equal a given Line.

LXIII.

To determine the Triangle, there are given, the Sum, or the Difference, of two Lines, to which the Base, and a Line, drawn from the Vertex, making a given Angle with the Base, have given Ratios; also, two Lines, to which the Diameter of the inscribed Circle, and the Difference between the Sum of the Sides and the Base, have given Ratios.

LXIV.

Having a Point and a right Line, given in position: to draw two Lines, from the given point, including a given Angle, that shall cut off, from the Line given in position, two Segments, between the points of Intersection, and a given Point, or given Points, in that Line, the Rectangle of which shall be given.

draw GR parallel to a Line given in position; so, that adding a given Line to each of the Segments GR, GB, GP; or, taking a given Line from each of them; the Rectangle under two of the Sums, or under two of the Differences, shall have a given Ratio to the Square of the remaining Sum, or Difference.

Or so, that

adding a given Line to GR, GB, and taking a given Line from GP; or taking a given Line from GR, GB, and adding a given Line to GP: the Rectangle of the Sums, or of the Differences, shall have a given Ratio to the Square of the Difference, or of the Sum, of GP, and the given Line.

LXXI.

The Lines RN, DL, BM, and the Points P, R, Q, are given in position: to draw the Parallels PN, RL, meeting BN, DL, at N, L; so, that NL, and QM, parallel to NL, being drawn; the Lines PN, QM, shall meet BM, at the same Point M.

LXXII.

To determine the Triangle, there are given, the Base, a Line from the Vertex, dividing the Base

Circle; such, that adding to, or taking from, PR, QR, the given Lines PL, QK; or adding PL to one, and taking QK from the other; the Sums, or the Differences; or the Sum and Difference, shall have to each other a given Ratio.

LXVIII.

From two given Points, to draw two Lines, including a given Angle, and meeting two Lines given in position; so, that drawing, from the points of Intersection, two Lines, parallel to two other Lines given in position; the two Lines, last drawn, shall have their Point of concourse, in another right Line given in position.

LXIX.

The Lines FR, LP, KH, HF, are given in position: to draw GR, parallel to another Line given in position, meeting FR, LP, KH, HF, at R, P, B, G; so, that the Sum, or the Difference, of two Lines, to which GR, GB, have given Ratios, shall have a given Ratio to the Segment GP.

LXX.

The Lines are posited as in the preceding: to

draw GR parallel to a Line given in position; so, that adding a given Line to each of the Segments GR, GB, GP; or, taking a given Line from each of them; the Rectangle under two of the Sums, or under two of the Differences, shall have a given Ratio to the Square of the remaining Sum, or Difference.

Or so, that

adding a given Line to GR, GB, and taking a given Line from GP; or taking a given Line from GR, GB, and adding a given Line to GP: the Rectangle of the Sums, or of the Differences, shall have a given Ratio to the Square of the Difference, or of the Sum, of GP, and the given Line.

LXXI.

The Lines RN, DL, BM, and the Points P, R, Q, are given in position: to draw the Parallels PN, RL, meeting BN, DL, at N, L; so, that NL, and QM, parallel to NL, being drawn; the Lines PN, QM, shall meet BM, at the same Point M.

LXXII.

To determine the Triangle, there are given, the Base, a Line from the Vertex, dividing the Base

Circle; such, that adding to, or taking from, PR, QR, the given Lines PL, QK; or adding PL to one, and taking QK from the other; the Sums, or the Differences; or the Sum and Difference, shall have to each other a given Ratio.

LXVIII.

From two given Points, to draw two Lines, including a given Angle, and meeting two Lines given in position; so, that drawing, from the points of Intersection, two Lines, parallel to two other Lines given in position; the two Lines, last drawn, shall have their Point of concourse, in another right Line given in position.

LXIX.

The Lines FR, LP, KH, HF, are given in position: to draw GR, parallel to another Line given in position, meeting FR, LP, KH, HF, at R, P, B, G; so, that the Sum, or the Difference, of two Lines, to which GR, GB, have given Ratios, shall have a given Ratio to the Segment GP.

LXX.

The Lines are posited as in the preceding: to

draw GR parallel to a Line given in position; so, that adding a given Line to each of the Segments GR, GB, GP; or, taking a given Line from each of them; the Rectangle under two of the Sums, or under two of the Differences, shall have a given Ratio to the Square of the remaining Sum, or Difference.

Or so, that

adding a given Line to GR, GB, and taking a given Line from GP; or taking a given Line from GR, GB, and adding a given Line to GP: the Rectangle of the Sums, or of the Differences, shall have a given Ratio to the Square of the Difference, or of the Sum, of GP, and the given Line.

LXXI.

The Lines RN, DL, BM, and the Points P, R, Q, are given in position: to draw the Parallels PN, RL, meeting BN, DL, at N, L; so, that NL, and QM, parallel to NL, being drawn; the Lines PN, QM, shall meet BM, at the same Point M.

LXXII.

To determine the Triangle, there are given, the Base, a Line from the Vertex, dividing the Base

in a given Ratio, and the Sum, or the Difference, of two Lines, to which the Sides have given Ratios.

LXXIII.

Given, the Base, a Line from the Vertex, to a given Point in the Base, and the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the Sides have given Ratios, to determine the Triangle.

LXXIV.

Given, the Difference of the Angles at the Base, a Line, to which the Sum, or the Difference, of the Sides, has a given Ratio, and a Line, having a given Ratio, to the Sum, or to the Difference, of two Lines, one of which has a given Ratio to a Line drawn from the Vertex, making a given Angle with the Base; and the other, a given Ratio to the Difference of the Segments of the Base, by a Perpendicular from the vertical Angle.

LXXV.

The Lines DR, KNM, and the Points P, Q, are given in position: to determine the Point R, in DR; such, that drawing RP, RQ; also PM, QN, making RPM, RQN, equal to two given Angles; the Ratio, Sum, or Difference, of the Segments KM, KN, shall be given.

LXXVI.

Given, in position, two Points, and three right Lines: to assign the position of a Point, in one of the Lines given in position; such, that, thence, drawing Lines to the two given Points, and, from those points, Lines making given Angles with the Lines first drawn: the Lines, last drawn, shall meet the two remaining Lines given in position, and cut therefrom, two Segments, of which, the Ratio, Sum, or Difference, shall be given.

LXXVII.

Given in position, the three Points P, Q, R, and the Lines DV, HKL: to draw PV, QV, to intersect at V, in DV; so, that drawing PL, RK, making given Angles with VP, VQ, meeting HL at L, K; the Ratio of the Segments HL, HK: or the Sum, or Difference, of Lines, to which HL, HK, have given Ratios; shall be given.

Line QR; and H, in BD; are given: to draw PE, QE, to intersect in BE, so, that from D, the Point of Intersection of PE, BD, drawing DI parallel to EQ, meeting QR, at I: the Sum, or the Difference, of HD, RI; or the Sum, or the Difference, of two Lines, to which HD, RI, have given Ratios, shall be given.

LVII.

Given the Ratio of the Segments of the Base, by a Line from the Vertex, making a given Angle with the Base, the Sum, or the Difference, of two Lines, to which the Sides have given Ratios, and the vertical Angle; to determine the Triangle.

LVIII.

The right Lines HF, FN, DQ, BK, and the Points H, I, in HF, NF; are given in position: to draw NQK, parallel to another Line given in position, meeting FN, DQ, BK, at N, Q, K; so, that making HM : NI, and FV : FM, equal to given Ratios, and joining K, V:

A Parallel to KV, through Q, shall pass through a given Point.

LIX.

The Lines EQ, DR, FM, LN, and the Point

P, are given in position: to draw QR, parallel to another Line given in position, meeting EQ, DR, at Q, R; so, that drawing QP, and applying, to FM, LN, the Lines RM, MN, having given Ratios to PQ; the Angles that PQ makes with RM, MN, shall be given in Magnitude.

LX.

To determine a Rhomboides, such, that its Sides shall pass through four given Points, and two of its opposite, angular Points, be posited upon two right Lines, given in position.

LXI.

Three Circles being given in position and Magnitude: to describe a fourth Circle, intersecting the given ones; so, that two of the Chords shall pass through two given Points, and the remaining Chord shall cut off Segments, from two Lines given in position; the Ratio, Sum, or Difference, of which, shall be given.

LXII.

Three Circles are given in position and Magnitude: to describe a fourth Circle, to intersect the given ones; so, that one of the Chords shall pass through a given Point, another shall make a given Angle with that passing through the given point, and the remaining Chord shall cut off, from two Lines given in position, two Segments, making the Sum, or the Difference, of two Lines, to which those Segments have given Ratios, equal a given Line.

LXIII.

To determine the Triangle, there are given, the Sum, or the Difference, of two Lines, to which the Base, and a Line, drawn from the Vertex, making a given Angle with the Base, have given Ratios; also, two Lines, to which the Diameter of the inscribed Circle, and the Difference between the Sum of the Sides and the Base, have given Ratios.

LXIV.

Having a Point and a right Line, given in position: to draw two Lines, from the given point, including a given Angle, that shall cut off, from the Line given in position, two Segments, between the points of Intersection, and a given Point, or given Points, in that Line, the Rectangle of which shall be given.

LXV.

Two given Circles, a Diameter AB, of one of them, and a Chord KL, of the other, are given in position: to assign, in AB, the points P, Q, such, that AP : PB= AQ : QB, and, that PK, QL, shall intersect in the Periphery of the Circle KL.

LXVI.

Given in position, four right Lines: to assign the position of a Point, such, that drawing, from that Point, four right Lines, meeting those given in position, in given Angles; the Sum, or the Difference, of two Lines, to which two of the Lines so drawn, have given Ratios, shall be given: and, the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the two remaining Lines, drawn from the point assigned, have given Ratios, equal a given Space.

LXVII.

To draw, from the Points P, Q, posited in the Diameter AB, of a given Circle, the Lines PR, QR, to R, a Point in the Periphery of the given

LXXXIV.

To draw, from one Extremity of the Diameter, of a given Circle, a Line that shall have a given Segment intercepted, by the Periphery, and a Line given in position, at right Angles to the given Diameter.

LXXXV.

The Points A, B, and the Lines KA, KP, are given in position: to draw AP, BP, to P, a point in KP; such, that AP shall be a Tangent to the Circle passing through the points A, B, C.

The Point C being the Intersection of the Lines AK, BP.

LXXXVI.

Given the vertical Angle, a Line, from the vertical Angle, making a given Angle with the Base, and a Line, to which the Perimeter has a given Ratio; to determine the Triangle.

LXXXVII.

√ Given the Hypothenuse, and the Sum of the Squares of a Line, to which one of the Sides has a given Ratio, and the adjacent, or alternate, Segment of the Hypothenuse, by a Perpendicular from the right Angle; to determine the Triangle.

LXXXVIII.

The Points A, B, C, and the Line DE, are given in position: to inflect AD, BD, CD, to D, a Point in DE; such, that the Angle ADB shall be equal to CDE.

LXXXIX.

To determine the Triangle, there are given, the vertical Angle, the Sum of the Sides, and the Rectangle of the Base and the Difference of the Sides.

XC.

The Lines AB, AC, and the point N, are given in position: to draw NP, PQ, PR, meeting AB, AC, at P, Q, R; so, that the Angles NPQ, APR, may be given in Magnitude, and the Segment NR equal a given Line.

XCI.

It is required to draw, from G, a Point in the Periphery of a given Circle, the Line GRM, meeting a Chord DK, at R, and the Circle at M; so, that joining R, and a point B, given in position; the Rectangle GM . RB shall be given.

XCII.

The Lines DR, GT, TN, are given in position; to draw the Line GRN, meeting DR, TN, at R, N; so, that joining R, and B, a given point, in GT; the Ratio GN : RB, may be given.

XCIII.

Three Circles being given in position and Magnitude: to describe a fourth Circle, intersecting the given ones; so, that the Chords joining the Points of Intersection, shall pass through three given Points.

XCIV.

In a given Triangle, to inscribe another Triangle; such, that one of its Sides shall be given, another pass through a given Point, and the remaining Side, be parallel to a Line given in position.

XCV.

To describe a Circle, through two given Points, to touch a Circle, given in position and Magnitude.

XCVI.

To describe a Circle, through a given Point P, cutting PC, a Line given in position, in L, and touching another Line CR, at R; so, that the Line LR shall pass through a point B, given in position.

XCVII.

Given the vertical Angle, a Line, from the Vertex, bisecting the Base, and, either the Sum, or the Difference, of the Perpendicular, and a Line to which the Base has a given Ratio: to determine the Triangle.

XCVIII.

To determine the Triangle, there are given, the Line bisecting the vertical Angle, the Difference of the Angles at the Base, and the Length of a Line, from the Vertex, dividing the Base in a given Ratio.

XCIX.

Given the Radii of three Circles, to assign their position; such, that a Tangent, drawn to any two of them, shall touch the remaining Circle.

C.

In a given Triangle, to inscribe another Triangle; such, that one of its Sides shall pass through a given Point, another shall cut off, from two Lines given in position, Segments, the Ratio, Sum, or Difference, of which, shall be given, and the remaining Side shall be given in Length.

CI.

The Lines GH, QL, and the Points A, B, P, are given in position: to describe a Circle, through the Points A, B, intersecting GH, in G, H; so, that joining P, G, and drawing BH, cutting QL in L; the Ratio QL : PG may be given.

The Point Q, in QL, being given.

CII.

In the given Line AC, is the Point B: to determine another point P, in AC; such, that the Rectangle under PA, and a given Line, shall be equal to the Sum of the Rectangles under PB, PC, and two other given Lines.

CIII.

To draw a Line, parallel to a Line given in position, meeting three other Lines given in position; so, that the Sum, or the Difference, of the Rectangles, under two Lines, to which the intercepted Segments have given Ratios, and two given Lines, shall be given.

CIV.

The Lines PD, FQ, CR, are given in position; of which, PD, FQ, are parallel: to assign the Position of a point Q, in FQ; such, that drawing QA, QR, parallel to two other Lines given in position, meeting PD, CR, at A, R; joining Q, and P, a given Point in DP; and drawing AR: the Lines PQ, AR, shall either be parallel, or include a given Angle.

CV.

In DT, one of three Lines DT, PR, AV, given in position, to determine a Point T, such, that drawing TR, TV, making given Angles, with PR, AV: the Line RV, joining the points R, V; shall be given in Length.

CVI.

The Lines AB, BQ, DR, are given in position: to determine a point P, in AB; such, that drawing PQR, parallel to another Line given in position, meeting BQ, DR, at Q, R; the Rectangle under a given Line, and a Line, to which PR has a given Ratio, shall be equal, or have a given Ratio, to the Sum, or to the Difference, of the Rectangles, under two other given Lines, and two Lines, to which PQ, DR, have given Ratios.

The Point D, in DR, being given.

CVII.

To determine a Point, in a Line given in position; such, that drawing, thence, three Lines, making given Angles, with three Lines given in position; the Rectangle under a given Line, and a Line, to which, one of the Lines drawn from the Point determined, has a given Ratio, shall be equal, or have a given Ratio, to the Sum, or to the Difference, of the Rectangles, under two other given Lines, and two Lines, to which the remaining Lines drawn from the assigned point, have given Ratios.

CVIII.

Determine the General Problem of Apollonius, on Inclinations, by Problem I. Book I. Also, show that it is the same as PROBLEM II. BOOK I. and PROBLEM LXXXIV. BOOK III.

CIX.

✓ To assign the Position of a Tangent, to a given Circle; such, that the Sum of the Squares, of two Perpendiculars upon the Tangent, from two given Points in the Periphery, shall be given.

CX,

✓ The Data as in the preceding: to determine the position of the Tangent; such, that adding a given Line to, or taking a given Line from, each of the Perpendiculars; the Sum of the Squares, of the compound Lines, or of the remaining Lines, shall be given.

CXI.

From two given Points, equally distant from the Center of a given Circle: to draw two Lines, to a Point in the Periphery; such, that the Sum of the fourth Powers, of those Lines, shall be given.

CXII.

Determine PROBLEMS XVI. and XXI. BOOK I.

by using Problem I. Book I. as an auxiliary Problem.

CXIII.

Determine the XVII. and XXV. Book I. by the same Auxiliary.

CXIV.

Let AB, AC, be Lines given in position; B, C, given points in those Lines, and P, Q, given Points, any how posited; to draw PD, QE, including a given Angle, meeting AB, AC, at D, E; so, that the Sum, or the Difference, of the Rectangles, under two given Lines, and two Lines, to which the Segments BD, CE, have given Ratios, shall be equal a given Square.

CXV.

Given one Side, an adjacent Angle, and the Area, to determine the Trapezium; when the Side opposite the given one, passes through a given Point, and the Angle opposite the given Angle, is posited in a right Line given in position.

CXVI.

The Lines DM, KQ, HP, LN, are given in

position: to draw a Line, parallel to a fifth Line, given in position, meeting DM, KQ, HP, LN, at M, Q, P, N; so, that the Rectangles MQ.QN, and MP.PN, shall have a given Ratio.

CXVII.

Given the Base, and the vertical Angle, to determine the Triangle; when the Rectangle under a given Line, and a Line, to which the Diameter of the inscribed Circle, has a given Ratio, is equal, or has a given Ratio, to the Square of the Sum, or of the Difference, of the Sides.

CXVIII.

Given one Side, the Angles at its Extremities, and the Sum, or the Difference, of two Lines, to which the Sides adjacent to the given one, have given Ratios, to determine the Quadrilateral; when the remaining Side passes through a given Point.

CXIX.

The Radii AB, DE, of two given Circles, are parallel to each other: to determine the Points I, K, in AB, DE; such, that drawing IQ, KR, at right Angles to AB, DE, meeting the Circles AB, DE, at Q, R; the Ratio IQ : KR shall be given, and the Line IK pass through a given Point P.

CXX.

The Lines AD, AB, and PM, parallel to AB; also, the Points P, M, Q; are given in position: to draw the parallels PB, AL, meeting AB, MP, at B, L; so, that drawing BD parallel to another Line given in position; and making MK : ML equal a given Ratio: the Points D, K, Q, shall be in the same right Line.

CXXI.

To draw a Line, parallel to a Line given in position, intersecting three other Lines given in position; so, that the Rectangle of the intercepted Segments, shall be equal to a given Square.

CXXII.

To draw a Line, as in the preceding; so, that adding a given Line to, or taking a given Line from, each of the intercepted Segments; or adding a given Line to one, and taking a given Line from the other Segment: the Rectangle under the Sums, or the Differences; or, under the Sum and Difference, shall be given.

CXXIII.

In a given Trapezium, to inscribe another Quadrilateral; such, that two of its Sides shall pass through two given Points, and the two remaining Sides shall be parallel to two Lines given in position.

CXXIV.

To determine the same as in the preceding; when three Sides, of the inscribed Figure, pass through three given Points, and the remaining Side is parallel to a Line given in position.

CXXV.

To assign the Position of the inscribed Trapezium; when three of its Sides pass through given Points, and the remaining Side cuts off, from two Lines given in position, two Segments, of which, the Ratio, Sum, or Difference, is given.

CXXVI.

To determine the same as in the above; when the Sides of the inscribed Quadrilateral pass through four given Points.

CXXVII.

Given the Perimeter, the Side of the inscribed Square, and the vertical Angle: to determine the Triangle.

CXXVIII.

The Lines AB, FD, KL; and the Points P, Q, R, K; are given in position: to draw PB, RL, parallel to each other, or to include a given Angle, meeting AB, KL, at B, L; so, that making KM : KL equal a given Ratio, and drawing QD, through M, meeting FD, at D: the Line BD shall either be parallel to, or make a given Angle with, another Line given in position.

CXXIX.

The Lines AG, AB, RD, MF, MK, are given in position: to assign the Position of a point P, in one of the Lines, as AG; such, that drawing PBD, PFK; parallel to two other Lines, also given in position; meeting AB, RD, MF, MK, at the points B, D, F, K: the Lines BF, DK, joining the Points of Intersection, shall include a given Angle.

CXXX.

The same Data, as in Problem CXXIX. to

determine P; such, that BF, DK, shall have their Point of concourse, in a right Line given in position.

CXXXI.

The Data remaining, as in the two preceding Problems: to assign the Point P; such, that BF, DK, shall intersect two other Lines, given in position, and cut therefrom, two Segments, between the Points of Intersection, and given Points therein; the Ratio, Sum, or Difference, of which, shall be given.

CXXXII.

The Lines DA, EB, FC, KP, LQ; and the Points K, L, in KP, LQ; are given in position: to describe a Triangle ABC, given in Species, having its angular Points, on three of the Lines DA, EB, FC, given in position; so, that the Segments KP, LQ, cut off, from KP, LQ, the two remaining Lines given in position, by one of the Sides, as AB, of the Triangle to be described, shall have a given Ratio.

CXXXIII.

The Data, as in the last Problem: to assign the Position of the Triangle ABC; such, that the

Sum, or the Difference, of two Lines, to which the Segments KP, LQ, have given Ratios, shall be given.

CXXXIV.

The Data remaining, as in the two preceding Problems: to determine the Position of the Triangle ABC; when right Lines, drawn from P, Q, to a given Point; or, to two Points, given in position; include a given Angle.

CXXXV.

Show, by Analysing, that all the Problems in Book I. may be determined by Problem I. Book I.

CXXXVI.

Show, in the same Way, that all the Problems in Book I. and Book II. may be determined by the General Problem.

END OF PART FIRST.

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